Circuits and Systems I

LECTURE #1

Sinusoids

Phase & Time-Shift

Complex Exponentials

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LIONS/Laboratory for Information and Inference Systems

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Outline - Today

- Today <> basics
  - Motivation + logistics
  - Chapter 2, pp. 9-17
  - Chapter 2, Sects. 2-3 to 2-5
- Next week <> Section 2-6
  Section 3-1
- Recommended self-study next week +
  - Chapter 1 ***read***
  - Appendix A: Complex Numbers ***read***
  - Appendix B: MATLAB ***skim***

Main Goal

- Students will be able to:

  Understand **mathematical** descriptions of signal processing **algorithms** and express those algorithms as computer **implementations** (e.g., via MATLAB)

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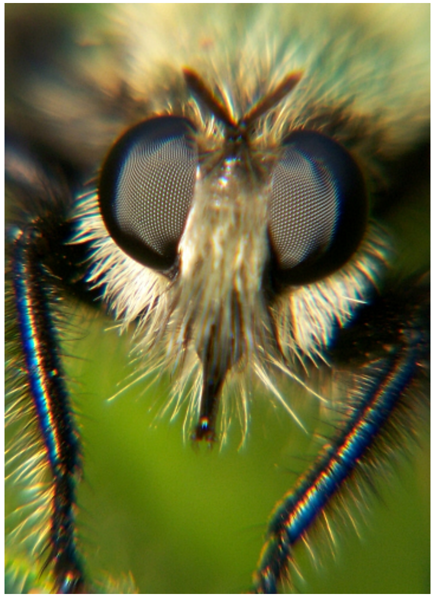
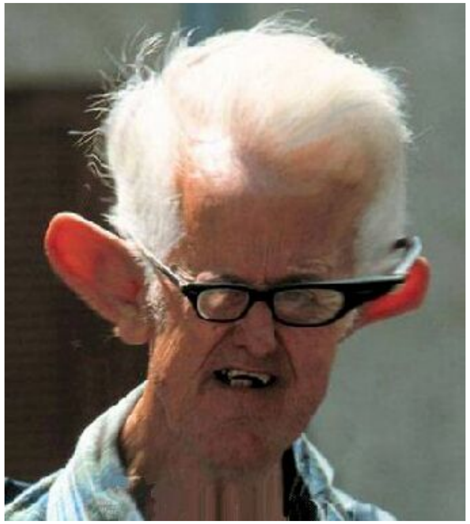
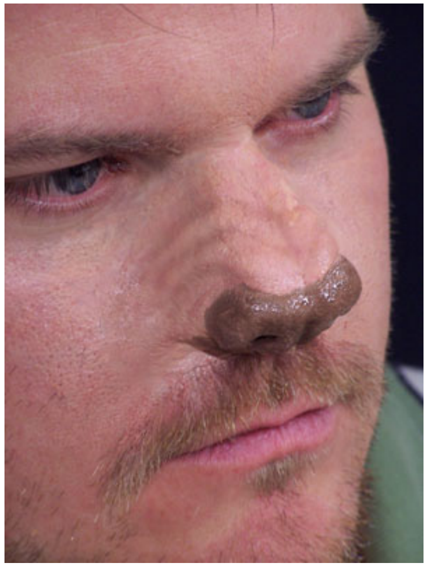




Sensing



Digital Revolution

12MP

25fps/1080p

4KHz

Multi touch

Digital Revolution

1977 – 5hours

12MP

25fps/1080p

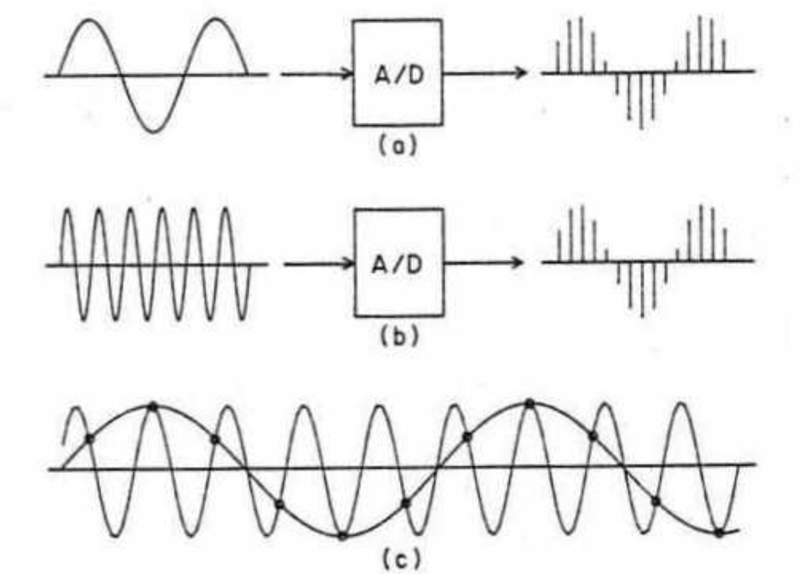
4KHz

<30mins

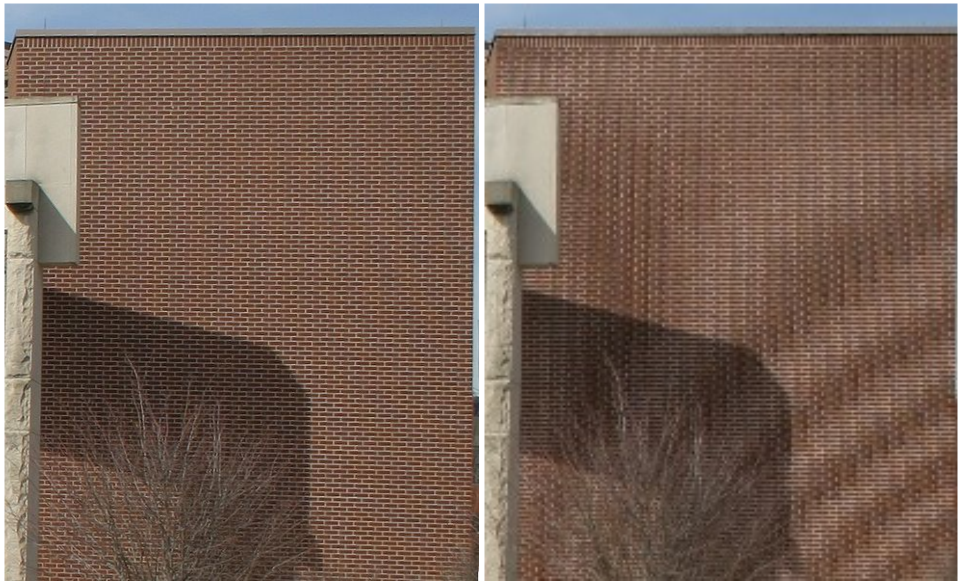
Digital Data Acquisition

Foundation: *Shannon/Nyquist sampling theorem*

"if you sample densely enough (at the Nyquist rate), you can perfectly reconstruct the original analog data"



time



space

Major Trends

higher resolution / **denser sampling**

12MP

25fps/1080p

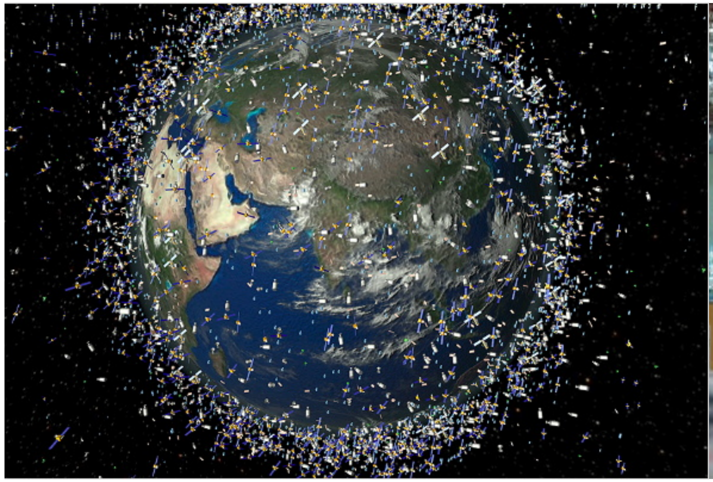
4KHz

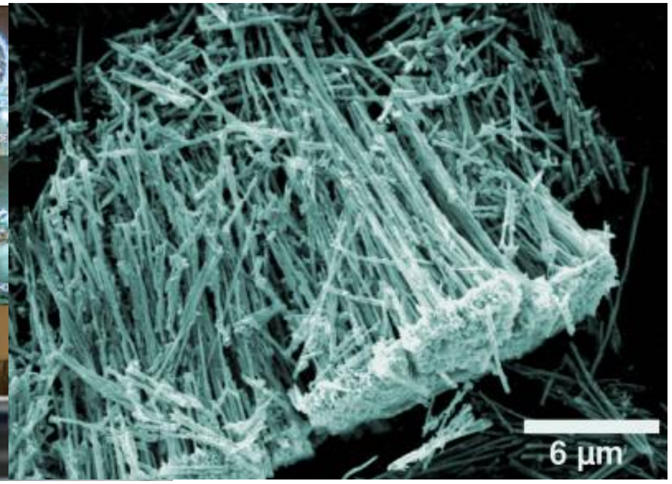
160MP

200,000fps

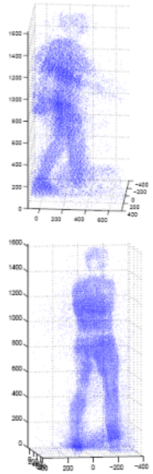
192,000Hz

Major Trends



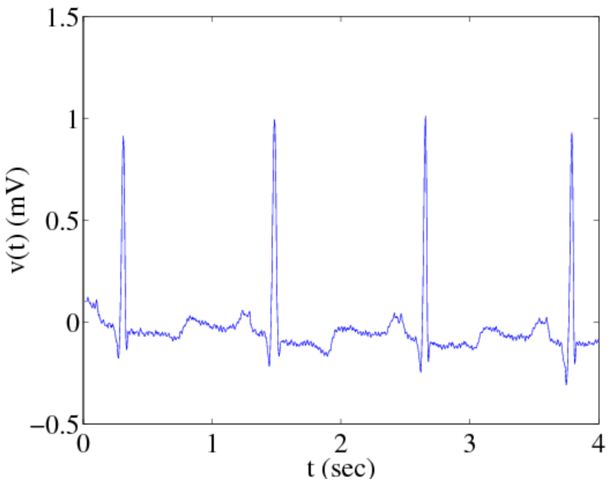


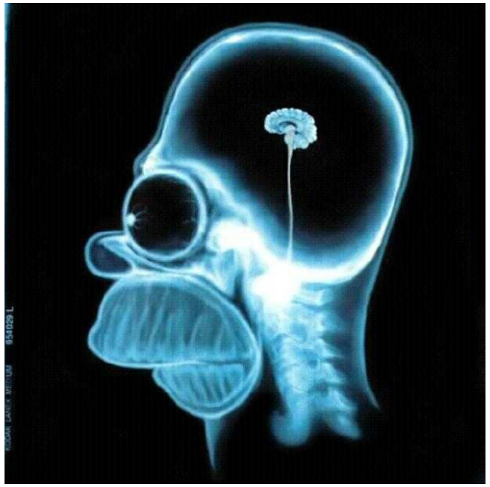
large numbers of sensors

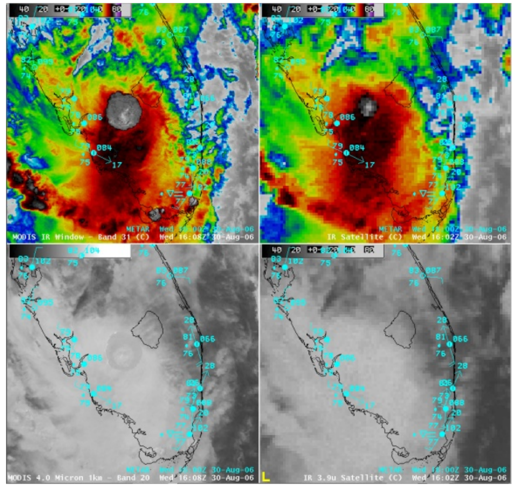


Major Trends

higher resolution / denser sampling
large numbers of sensors

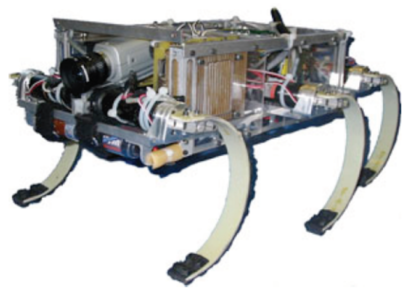






increasing # of modalities / **mobility**

acoustic, RF, visual, IR,
UV, x-ray, gamma ray, …



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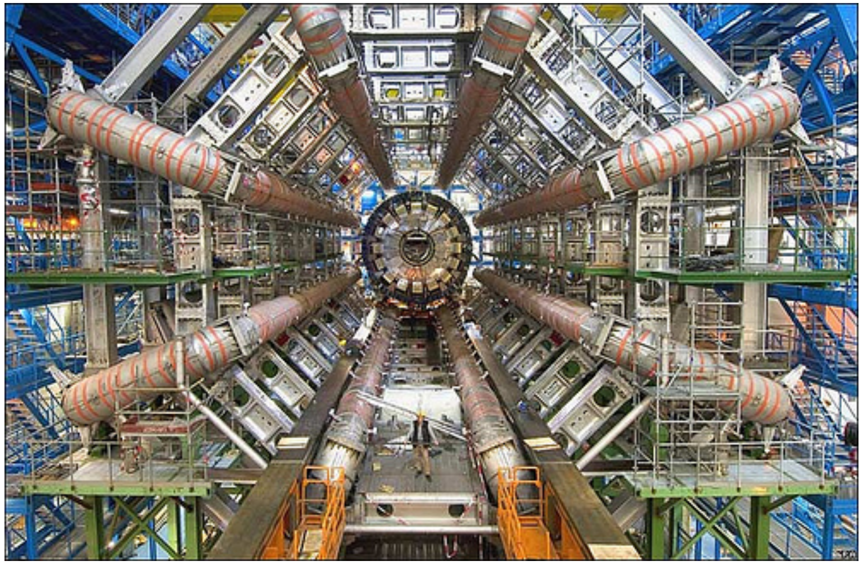
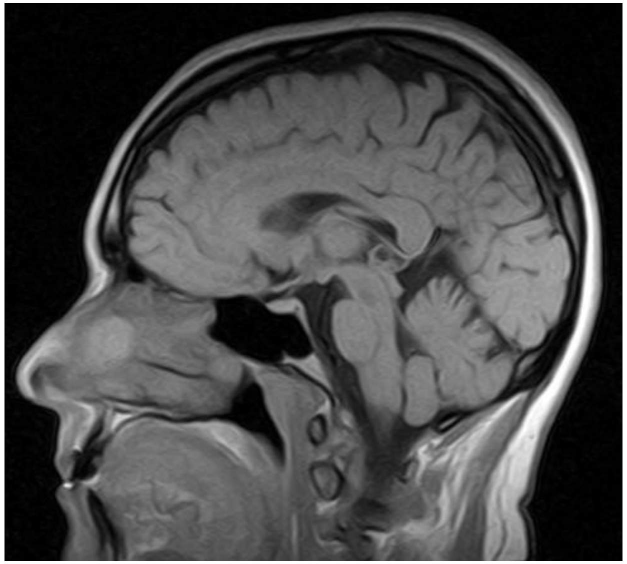




Motivation: solve bigger / more important problems

decrease acquisition times / costs

entertainment / new consumer products…

Midomi & Grooveshark

In General…

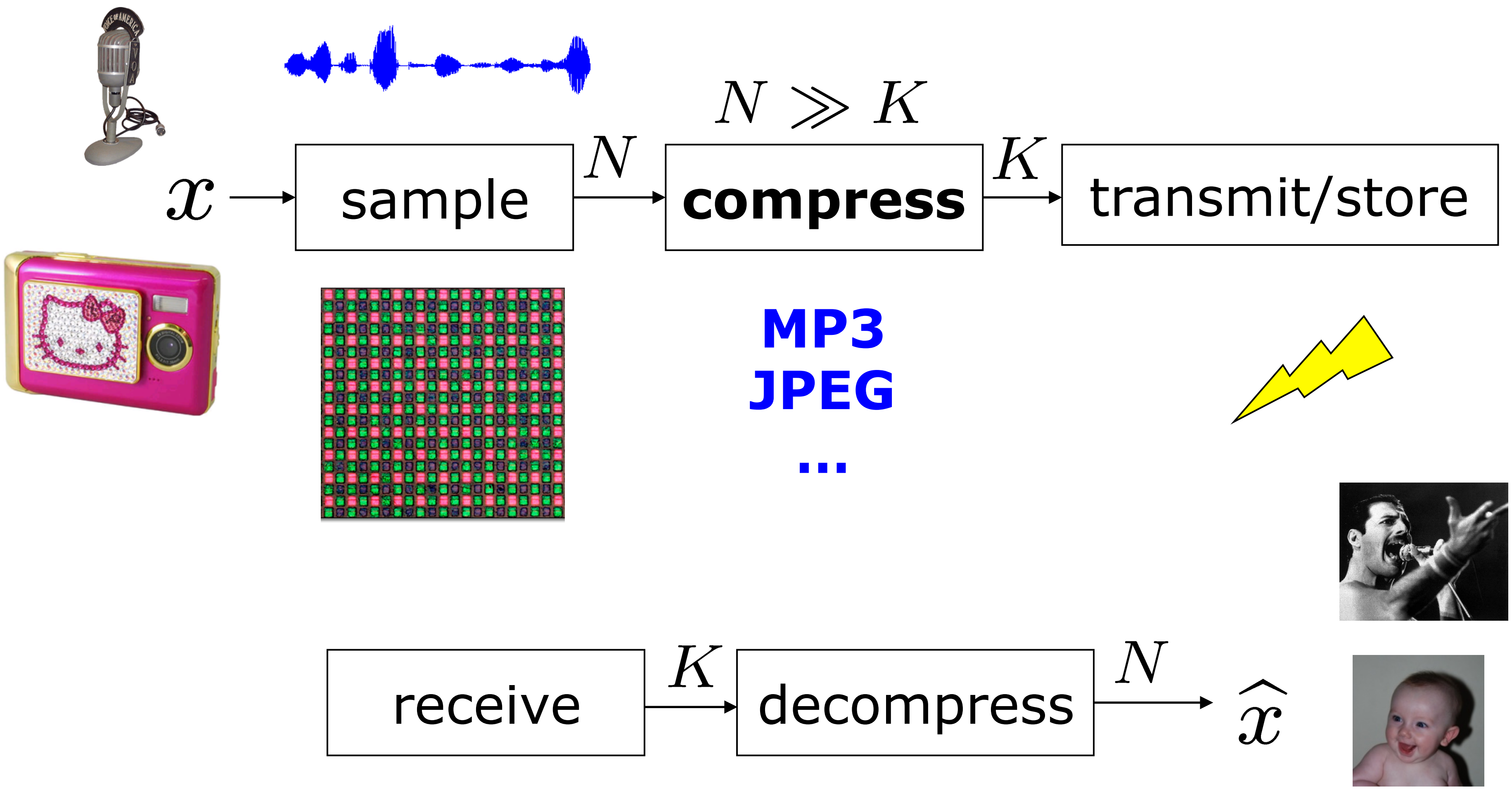
- Ignore
  - generalizes well
  - robust

Understanding the Basics

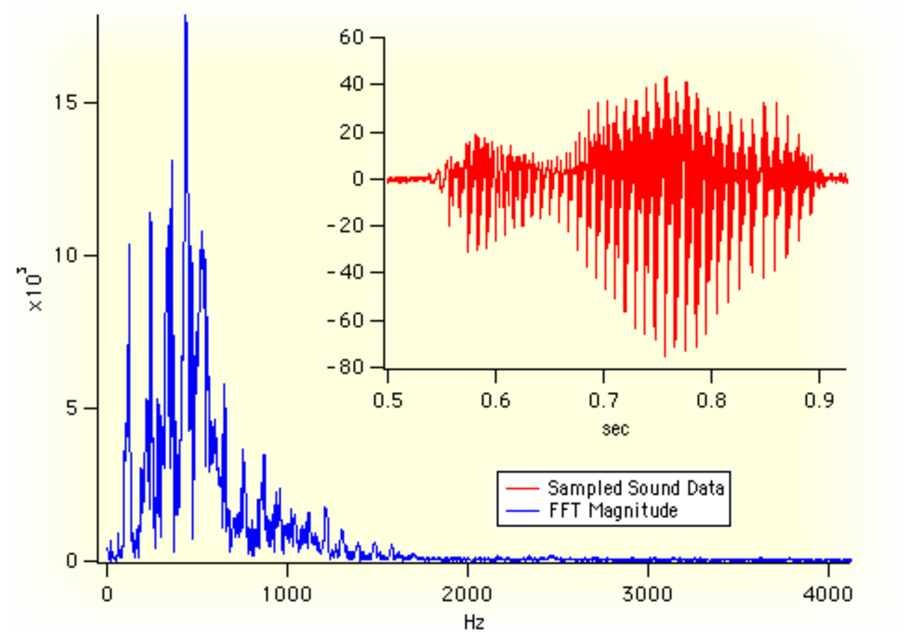
Circuits and Systems I

Sensing by *Sampling*

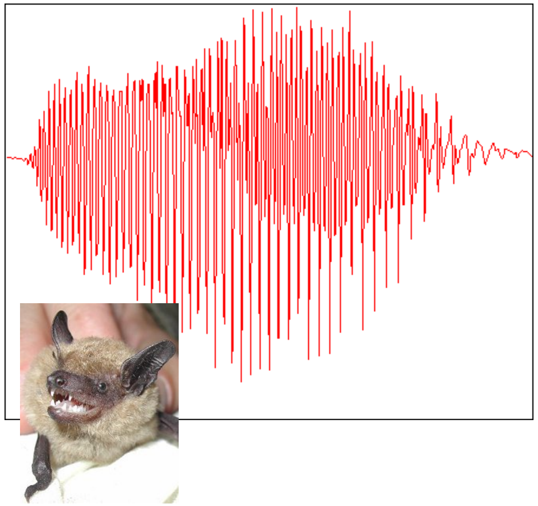
- Long-established paradigm for digital data acquisition
  - uniformly ***sample*** data at Nyquist rate (2x Fourier bandwidth)

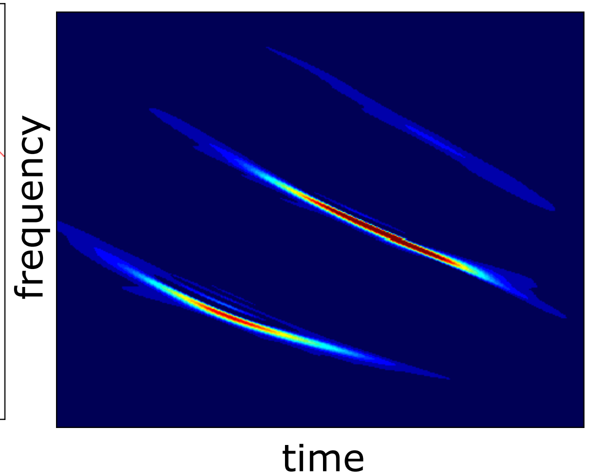


Transform Domain Representations



N
wideband signal samples

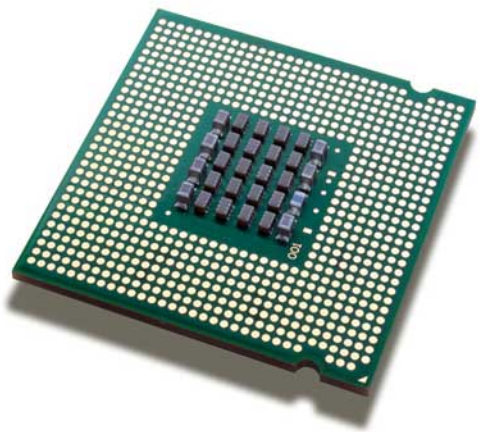


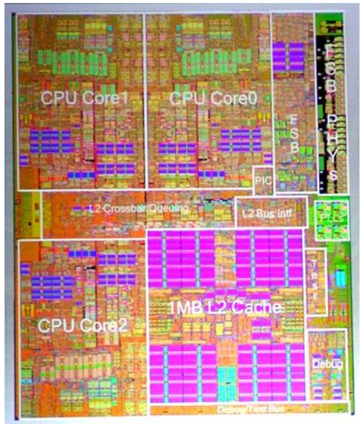


$K \ll N$
large time-frequency coefficients

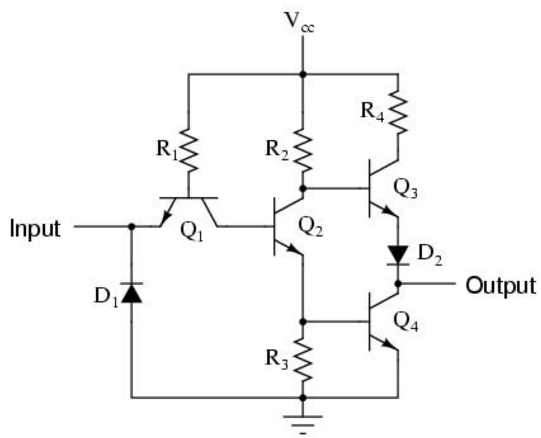
Processing by *Systems*

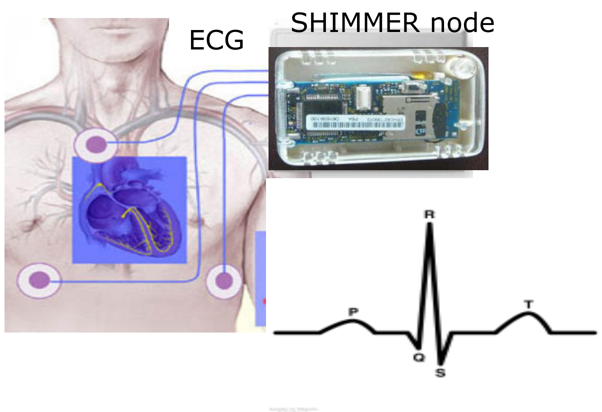
Signals > > > > Systems





Practical inverter (NOT) circuit





Sensor Cube
(with IMEC-NL)

Logistics

cf. Syllabus

Lecture Objectives

- Write general formula for a "sinusoidal" waveform, or signal
- From the formula, plot the sinusoid versus time
- What's a **signal**?
  - It's a **function** of time, $x(t)$
  - in the mathematical sense

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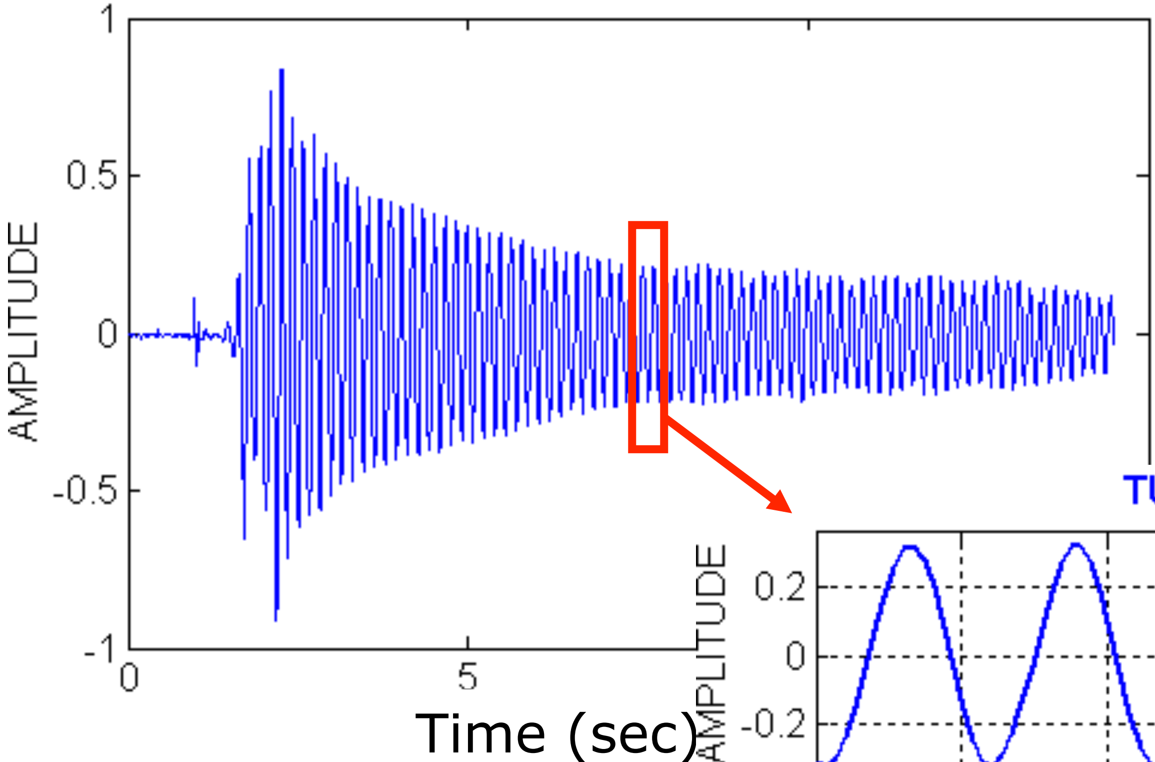
***CSI* Progress Level:**

Tuning Fork Example

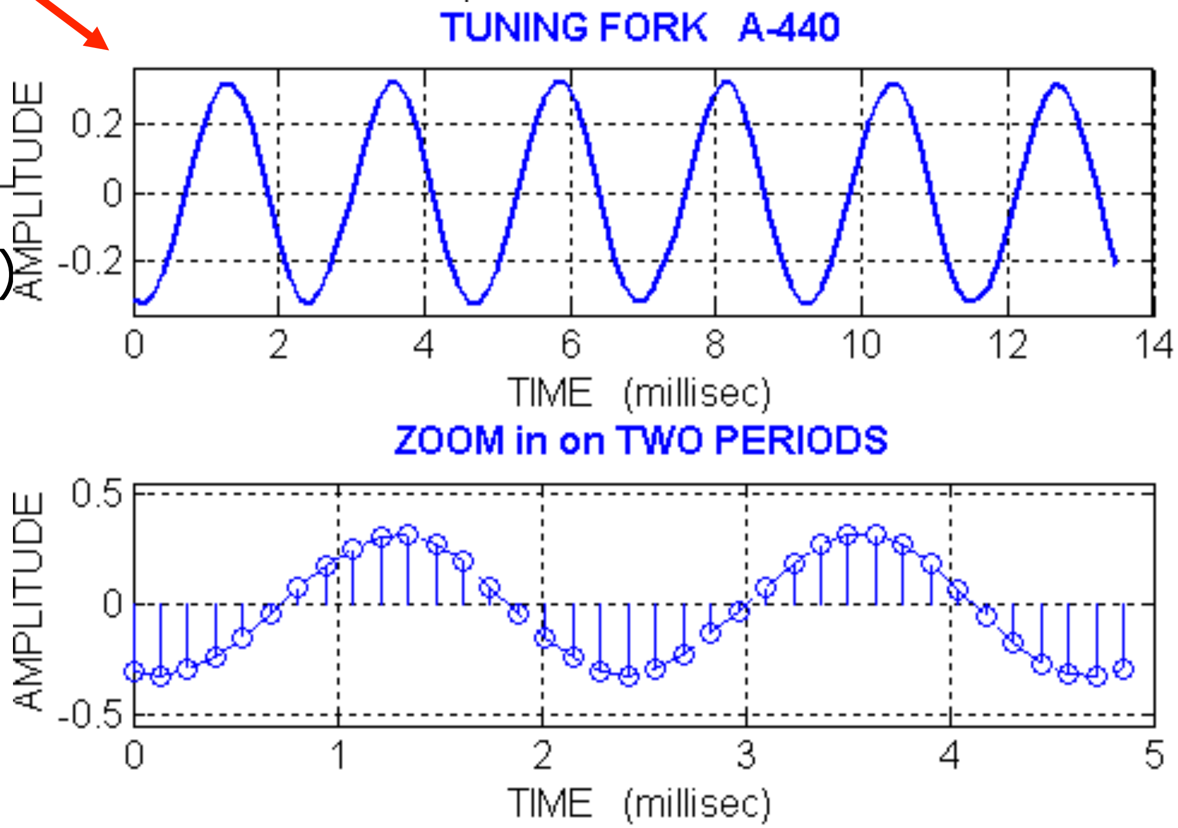
- CD-ROM demo
- “A” is at 440 Hertz (Hz)
- Waveform is a SINUSOIDAL SIGNAL
- Computer plot looks like a sine wave
- This should be the mathematical formula:

$$A\cos(2\pi(440)t+\varphi)$$

Tuning Fork A-440 Waveform



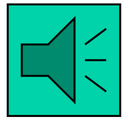
$$T \approx 8.15 - 5.85$$
$$= 2.3 \text{ ms}$$



$$f = 1/T$$
$$= 1000/2.3$$
$$\approx 435 \text{ Hz}$$

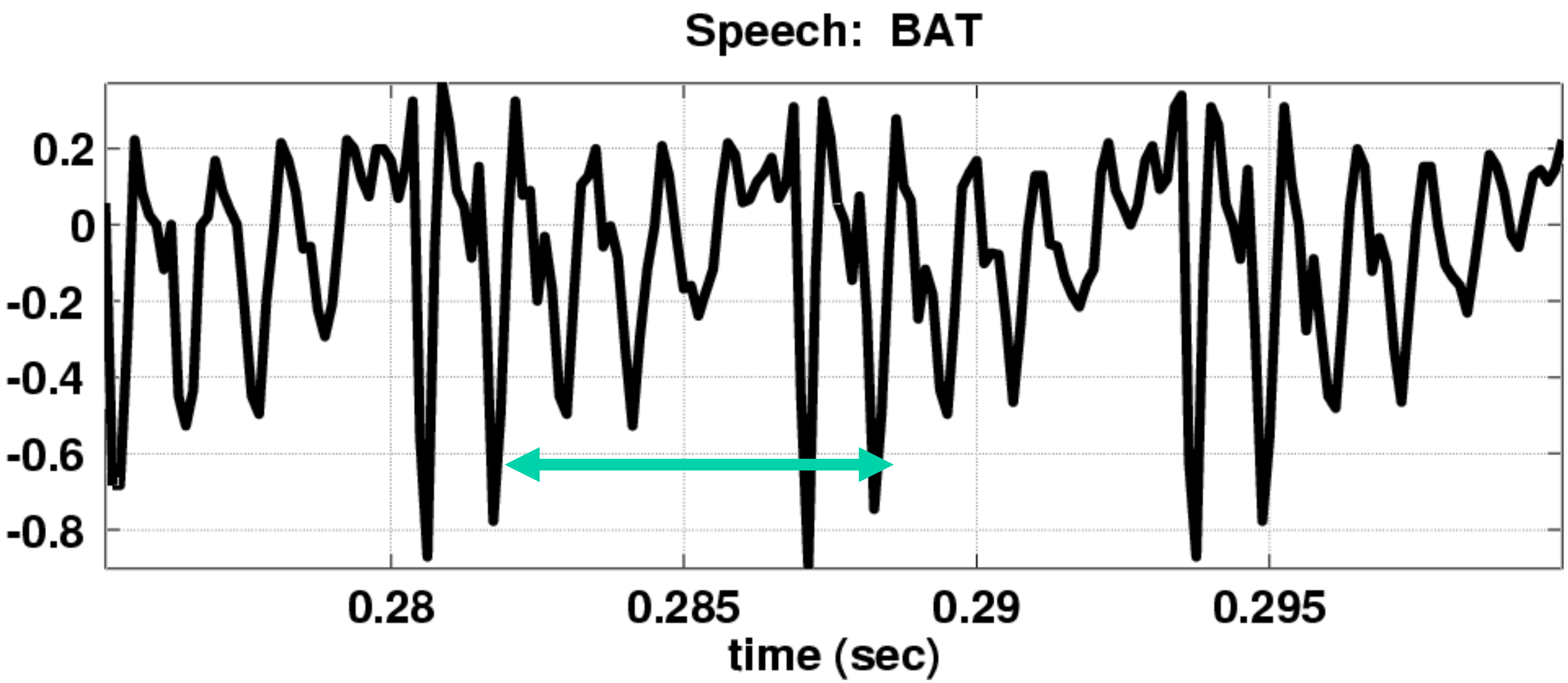
Speech Example

- More complicated signal (BAT.WAV)
- Waveform ***x*(*t*)** is NOT a Sinusoid
- Theory will tell us
  - ***x*(*t*)** is approximately a sum of sinusoids
  - FOURIER ANALYSIS
    - Break ***x*(*t*)** into its sinusoidal components
  - Called the FREQUENCY SPECTRUM



Speech Signal: BAT

- Nearly **Periodic** in Vowel Region
  - Period is (Approximately) T = 0.0065 sec



Digitize the Waveform

- ***x*[*n*]** is a SAMPLED SINUSOID
  - A list of numbers stored in memory
- Sample at 11,025 samples per second
  - Called the SAMPLING RATE of the A/D
  - Time between samples is
    - 1/11025 = 90.7 microsec
- Output via D/A hardware (at F_{samp})

Storing Digital Sounds

- ***x*[*n*]** is a SAMPLED SINUSOID
  - A list of numbers stored in memory
- CD rate is 44,100 samples per second
- 16-bit samples
- Stereo uses 2 channels
- Number of bytes for 1 minute is
  - 2 X (16/8) X 60 X 44100 = ?

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  - 2 X (16/8) X 60 X 44100 = 10.584 Mbytes

Sines and Cosines

- Always use the **cosine canonical form**

$$A\cos(2\pi(440)t + \varphi)$$

- Sine is a special case:

$$\sin(\omega t) = \cos(\omega t - \frac{\pi}{2})$$

Sinusoidal Signal

$$A\cos(\omega t+\varphi)$$

- **FREQUENCY** ω
  - Radians/sec
  - Hertz (cycles/sec)

$$\omega=(2\pi)f$$

- **AMPLITUDE** A
  - Magnitude

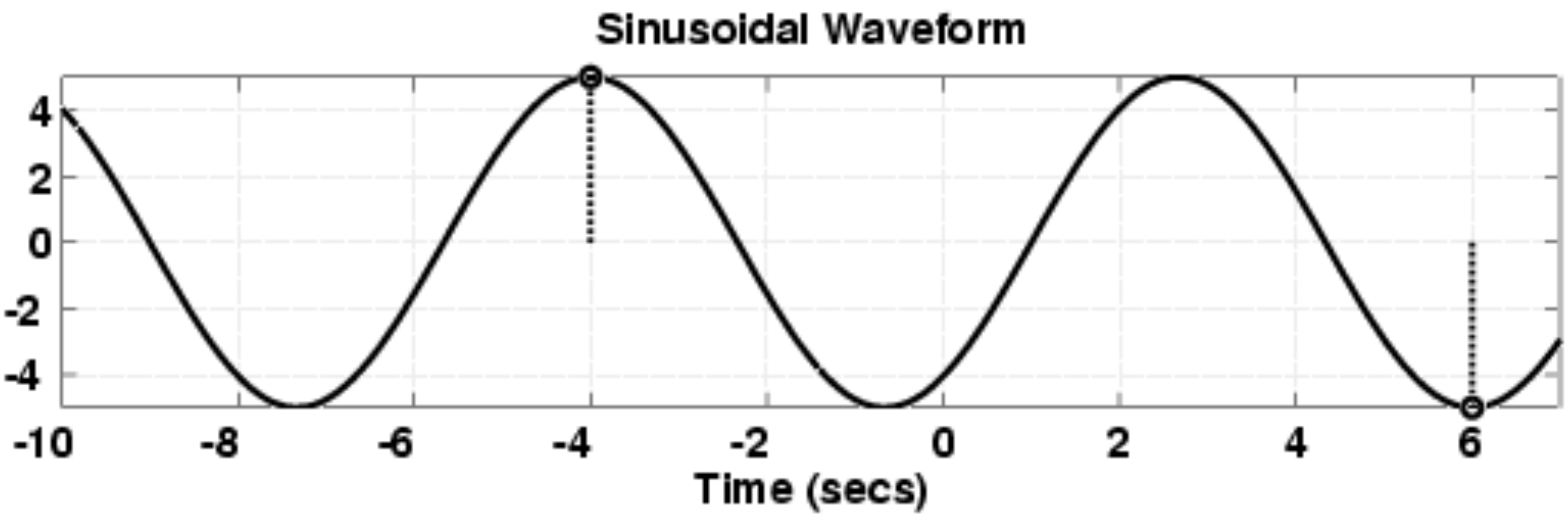
- **PERIOD** (in sec)

$$T=\frac{1}{f}=\frac{2\pi}{\omega}$$

- **PHASE** φ

Example Sinusoid

- Given the Formula $5\cos(0.3\pi t + 1.2\pi)$
- Make a plot



Plotting of a Cosine Signal

$$5\cos(0.3\pi t + 1.2\pi)$$

- Formula defines A, ω, and ϕ

$$A = 5$$

$$\omega = 0.3\pi$$

$$\varphi = 1.2\pi$$

Plotting of a Cosine Signal via the Mathematical Formula

$$5\cos(0.3\pi t + 1.2\pi)$$

- Determine **period**:

$$T = 2\pi / \omega = 2\pi / 0.3\pi = 20/3$$

- Determine a **peak** location by solving

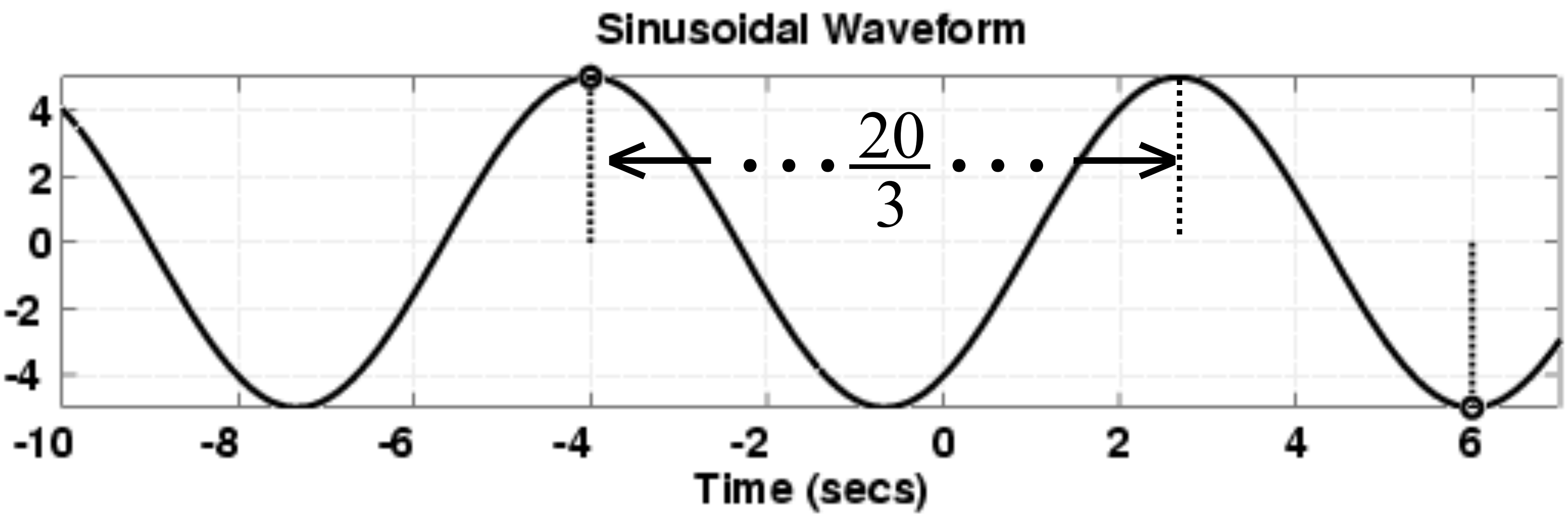
$$(\omega t + \varphi) = 0 \quad \Rightarrow (0.3\pi t + 1.2\pi) = 0$$

- **Zero** crossing is T/4 before or after
- Positive & Negative peaks spaced by T/2

Plotting

$$5\cos(0.3\pi t + 1.2\pi)$$

- Use T=20/3 and the peak location at t=-4



Lecture Objectives

- Define Sinusoid Formula from a plot
- Relate TIME-SHIFT to PHASE

Introduce an **ABSTRACTION**:

Complex Numbers **represent** Sinusoids

Complex Exponential Signal

$$z(t) = Xe^{j\omega t}$$

CSI **Progress Level:**

Time-Shift

- In a mathematical formula we can replace t with $t-t_m$

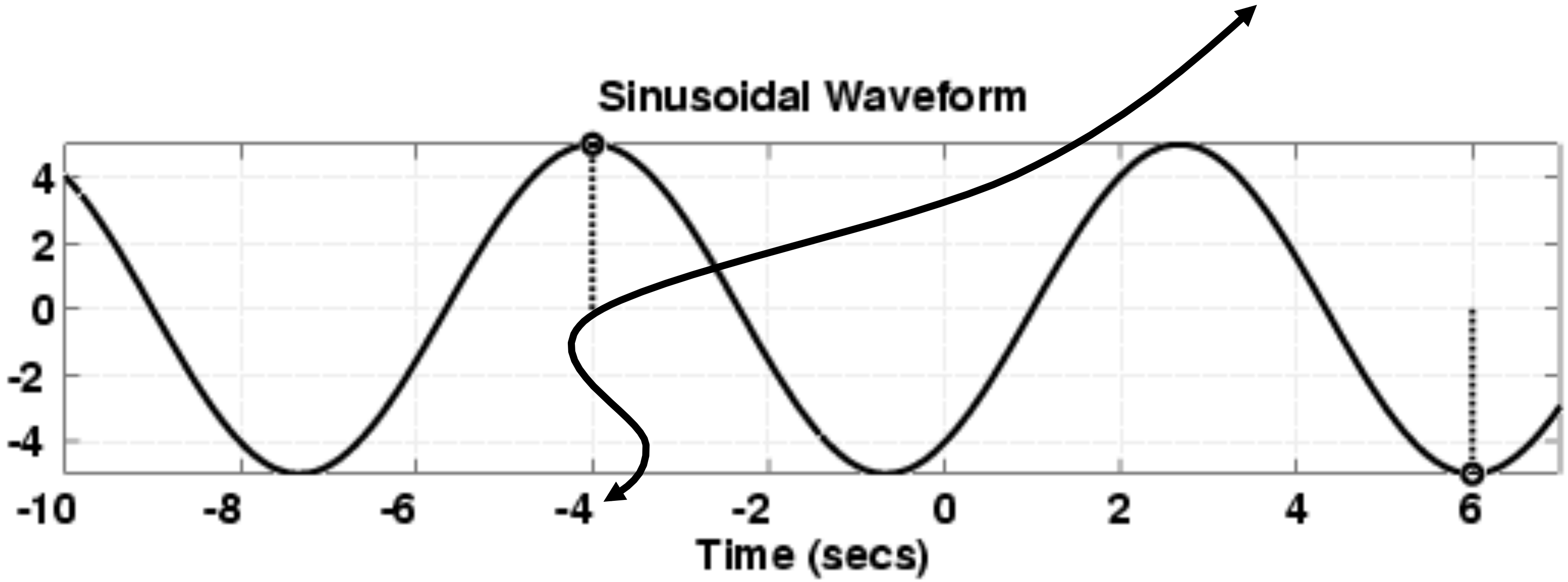
$$x(t-t_m) = A\cos(\omega(t-t_m))$$

- Then the $t=0$ point moves to $t=t_m$

- Peak value of $\cos(\omega(t-t_m))$ is now at $t=t_m$

TIME-SHIFTED SINUSOID

$$x(t+4) = 5\cos(0.3\pi(t+4)) = 5\cos(0.3\pi(t-(-4)))$$



Phase <> Time-Shift

- Equate the formulas:

$$A\cos(\omega(t - t_m)) = A\cos(\omega t + \varphi)$$

- and we obtain:

$$-\omega t_m = \varphi$$

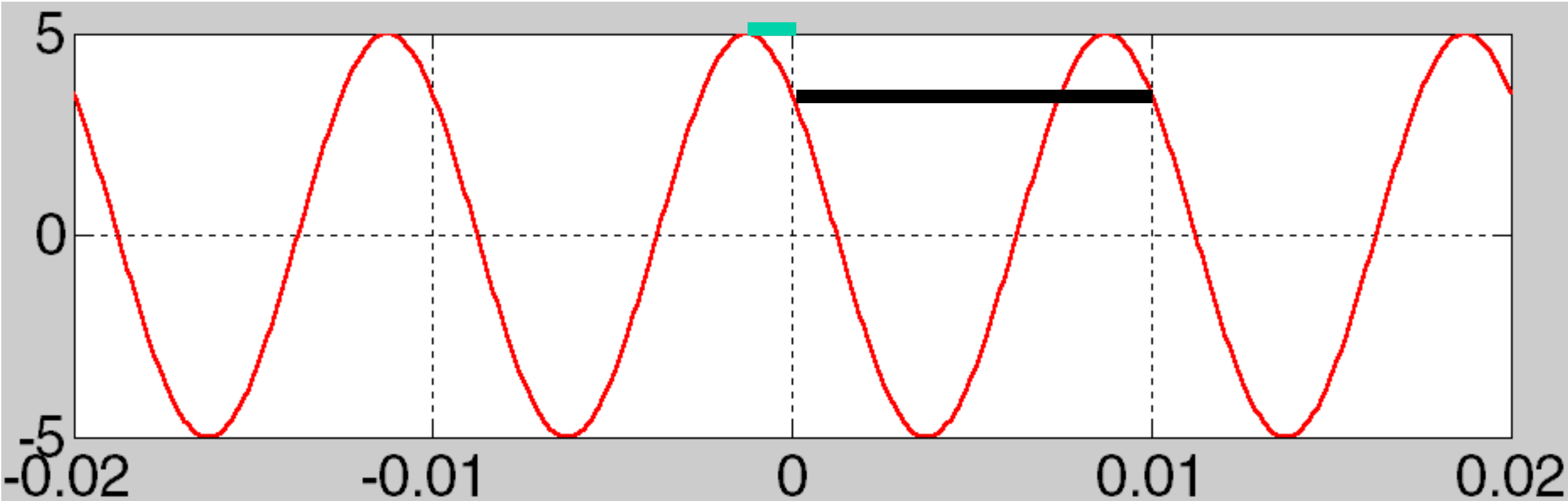
- or,

$$t_m = -\frac{\varphi}{\omega}$$

Sinusoid from a Plot

- Measure the period, T
  - Between peaks or zero crossings
  - Compute frequency: $\omega = 2\pi/T$
- Measure time of a peak: t_m
  - Compute phase: $\phi = -\omega\, t_m$
- Measure height of positive peak: A

(A, ω, φ) from a PLOT



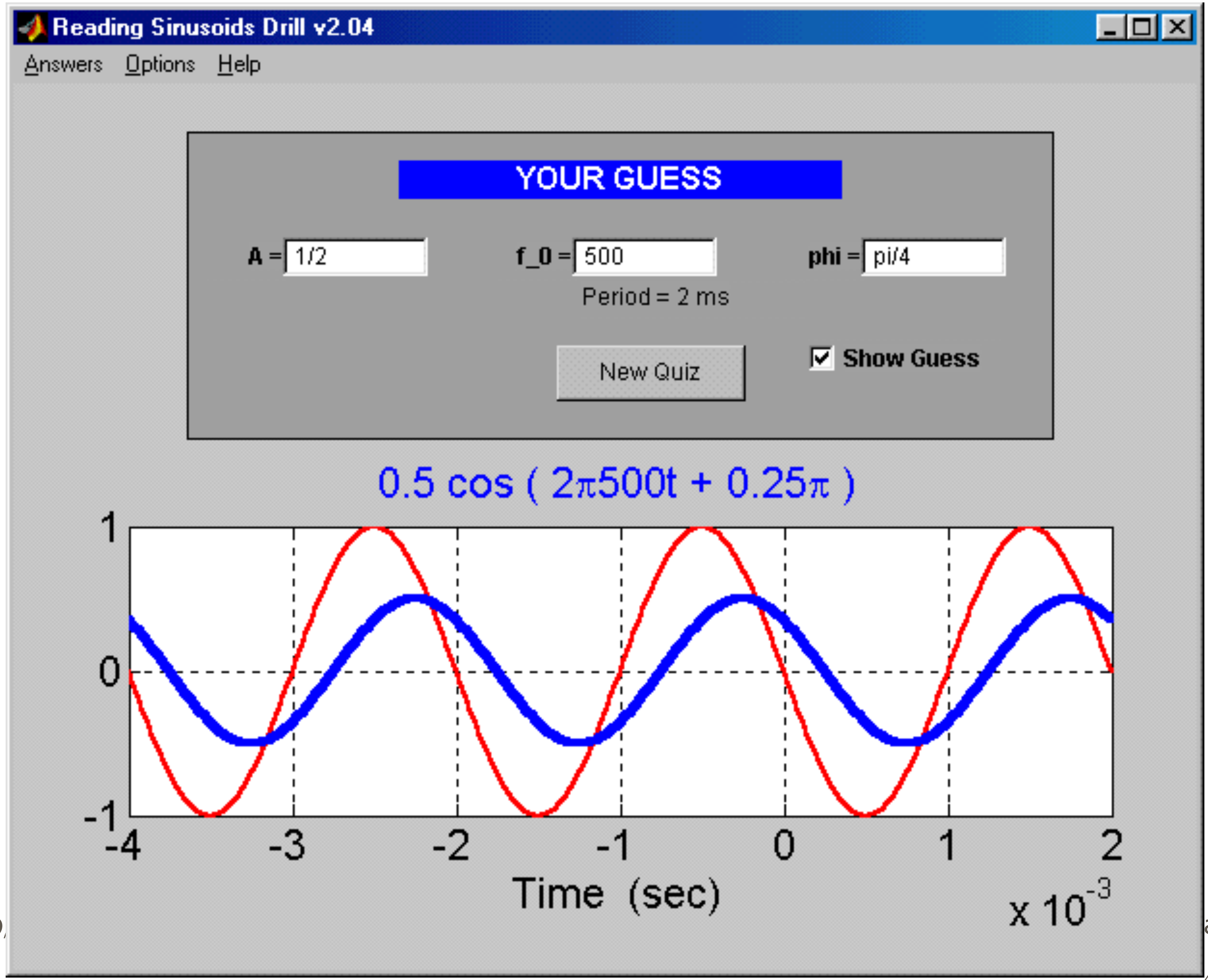
$$T = \frac{0.01\,\text{sec}}{1\,\text{period}} = \frac{1}{100}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.01} = 200\pi$$

$$t_m = -0.00125\,\text{sec}$$

$$\varphi = -\omega t_m = -(200\pi)(t_m) = 0.25\pi$$

Sine Drill (MATLAB GUI)



Phase is Ambiguous

- The cosine signal is periodic
  - Period is 2π

$$A\cos(\omega t + \varphi + 2\pi) = A\cos(\omega t + \varphi)$$

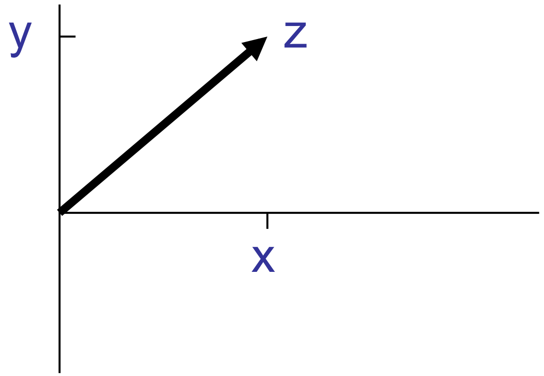
  - Thus adding any multiple of 2π leaves x(t) unchanged

$$\text{if } \; t_m = \frac{-\varphi}{\omega}, \; \text{then}$$

$$t_{m_2} = \frac{-(\varphi + 2\pi)}{\omega} = \frac{-\varphi}{\omega} - \frac{2\pi}{\omega} = t_m - T$$

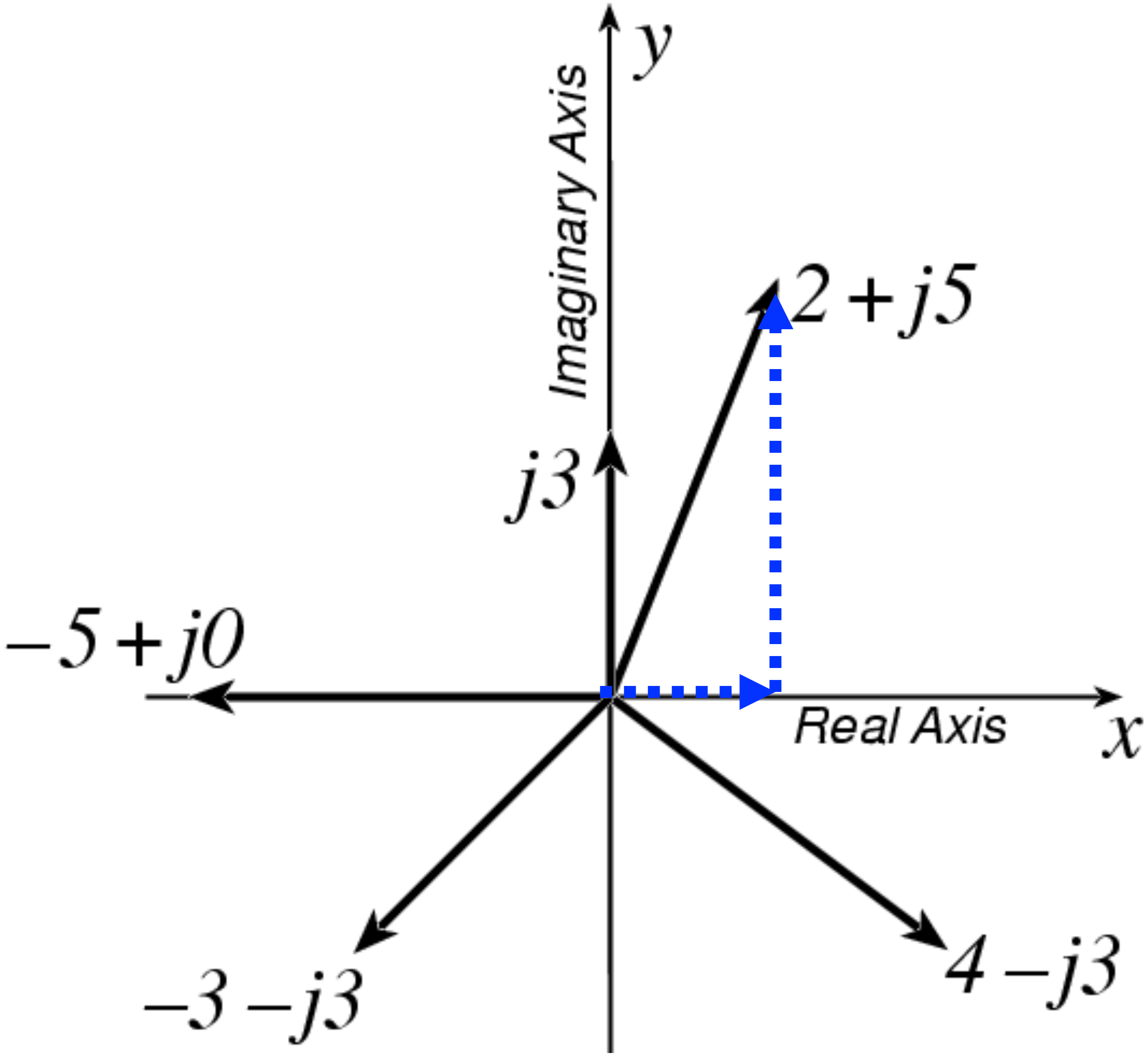
Complex Numbers

- To solve: $z^2 = -1$
  - $z = \boldsymbol{j}$
  - Math and Physics use $z = \boldsymbol{i}$
- Complex number: $z = x + \boldsymbol{j}\, y$

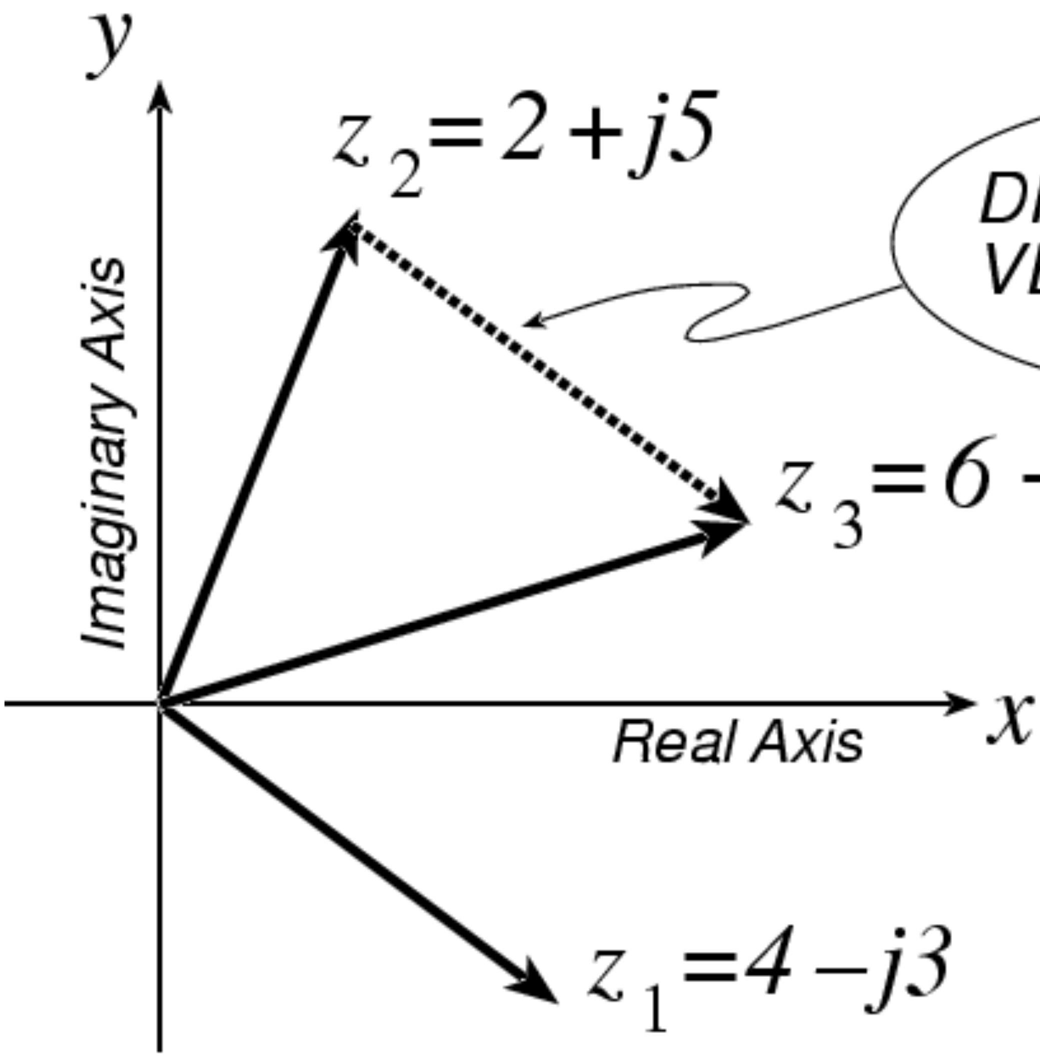


Cartesian coordinate system

Plot Complex Numbers



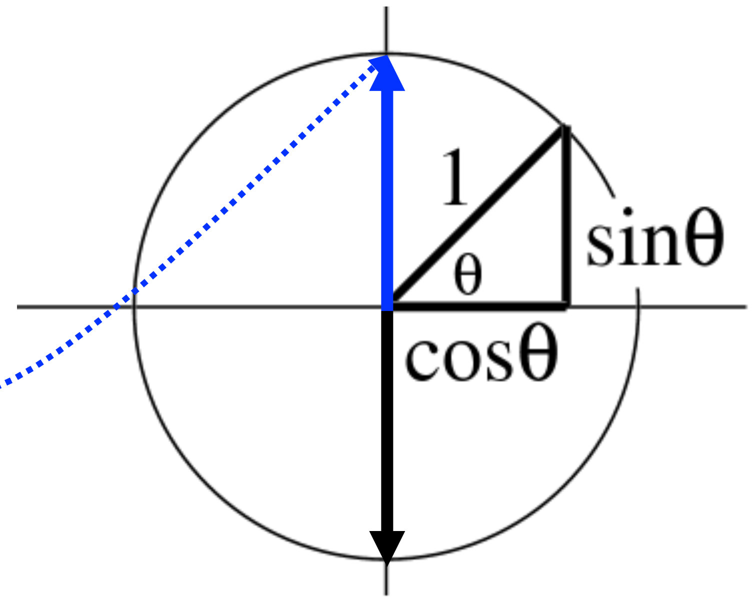
Complex Addition = **VECTOR** Addition



$$
\begin{aligned}
z_3 &= z_1 + z_2 \\
&= (4 - j3) + (2 + j5) \\
&= (4 + 2) + j(-3 + 5) \\
&= 6 + j2
\end{aligned}
$$

Polar Form

- Vector Form
  - **Length** =1
  - **Angle** = θ



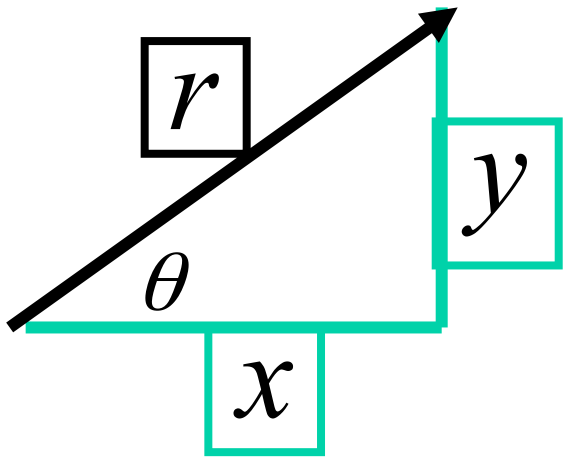
- Common Values
  - **j** has angle of 0.5π
  - -1 has angle of π
  - $-$**j** has angle of 1.5π
  - also, angle of **−j could** be $-0.5\pi = 1.5\pi - 2\pi$
  - because the PHASE is **AMBIGUOUS**

Polar <> Rectangular

- Relate (x,y) to (r,θ)

$$r^2 = x^2 + y^2$$

$$\theta = \mathrm{Tan}^{-1}\left(\frac{y}{x}\right)$$



$$x = r\cos\theta$$

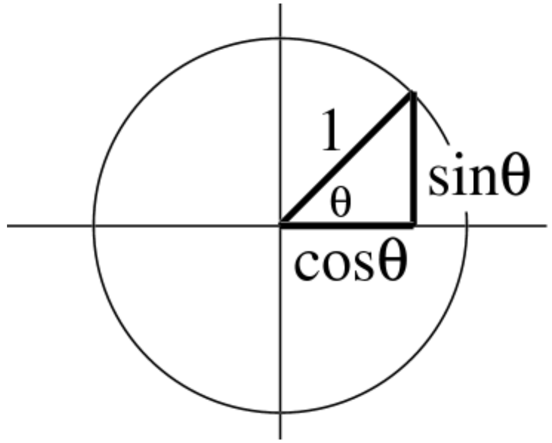
$$y = r\sin\theta$$

Most calculators do Polar-Rectangular

Need a notation for POLAR FORM

Euler's Formula

- **Complex Exponential**
  - Real part is cosine
  - Imaginary part is sine
  - Magnitude is one



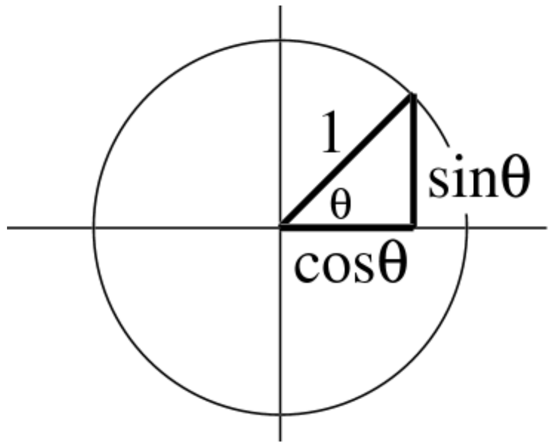
$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$re^{j\theta} = r\cos(\theta) + jr\sin(\theta)$$

Complex Exponential

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

- Interpret this as a **Rotating Vector**
  - $\theta = \omega \boldsymbol{t}$
  - Angle changes vs. time
  - ex: $\omega=20\pi$ rad/s
  - Rotates 0.2π in 0.01 secs



$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

cos = Real Part

Real Part of Euler's $\cos(\omega t) = \Re e\{e^{j\omega t}\}$

General Sinusoid $x(t) = A\cos(\omega t + \varphi)$

So, $A\cos(\omega t + \varphi) = \Re e\{Ae^{j(\omega t + \varphi)}\}$

$$= \Re e\{Ae^{j\varphi}e^{j\omega t}\}$$

Real Part Example

$$A\cos(\omega t+\varphi)=\Re e\left\{Ae^{j\varphi}e^{j\omega t}\right\}$$

Evaluate: $x(t)=\Re e\left\{-3je^{j\omega t}\right\}$

Answer:

$$x(t)=\Re e\left\{(-3j)e^{j\omega t}\right\}$$
$$=\Re e\left\{3e^{-j0.5\pi}e^{j\omega t}\right\}=3\cos(\omega t-0.5\pi)$$

Complex Amplitude

General Sinusoid

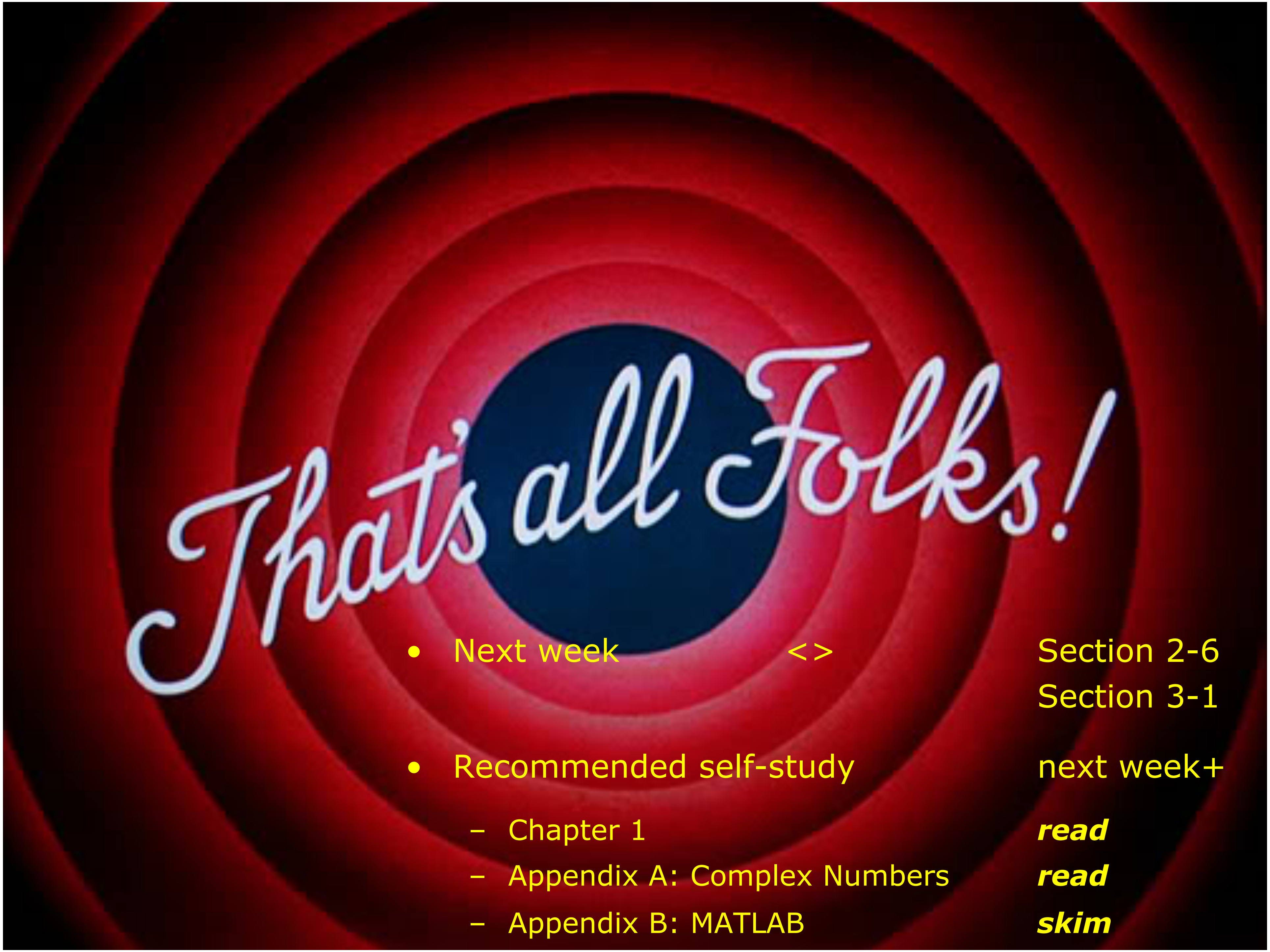
$$x(t) = A\cos(\omega t + \varphi) = \Re e\{Ae^{j\varphi}e^{j\omega t}\}$$

Complex AMPLITUDE = X

$$z(t) = Xe^{j\omega t} \qquad X = Ae^{j\varphi}$$

Then, any Sinusoid = REAL PART of $Xe^{j\omega t}$

$$x(t) = \Re e\{Xe^{j\omega t}\} = \Re e\{Ae^{j\varphi}e^{j\omega t}\}$$



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