ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Circuits and Systems I

LECTURE #10
FIR Filtering: An Introduction

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License Info for SPFirst Slides

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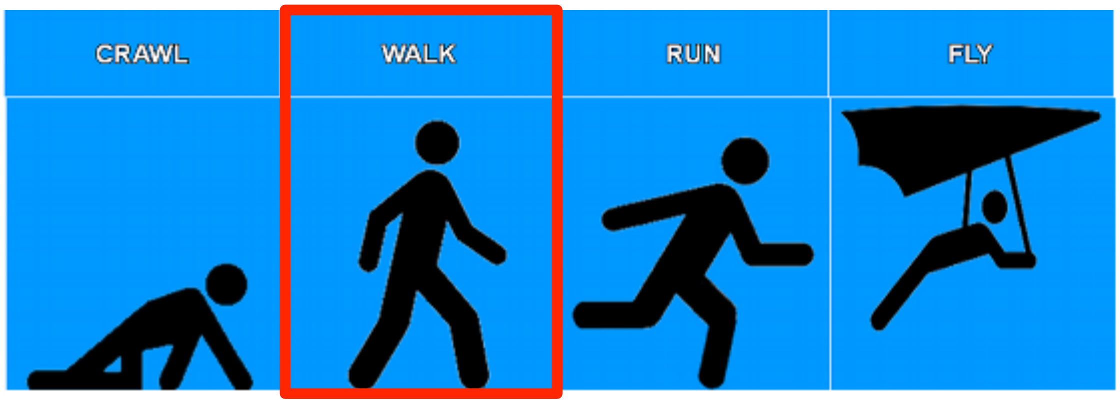
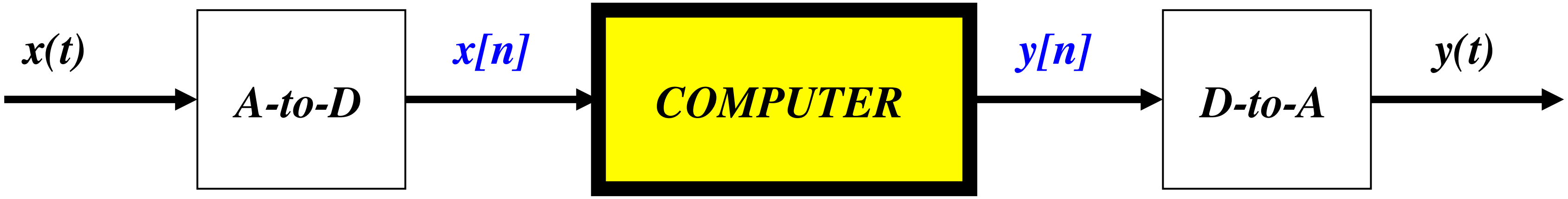
***CSI* Progress Level:**



Lecture Objectives

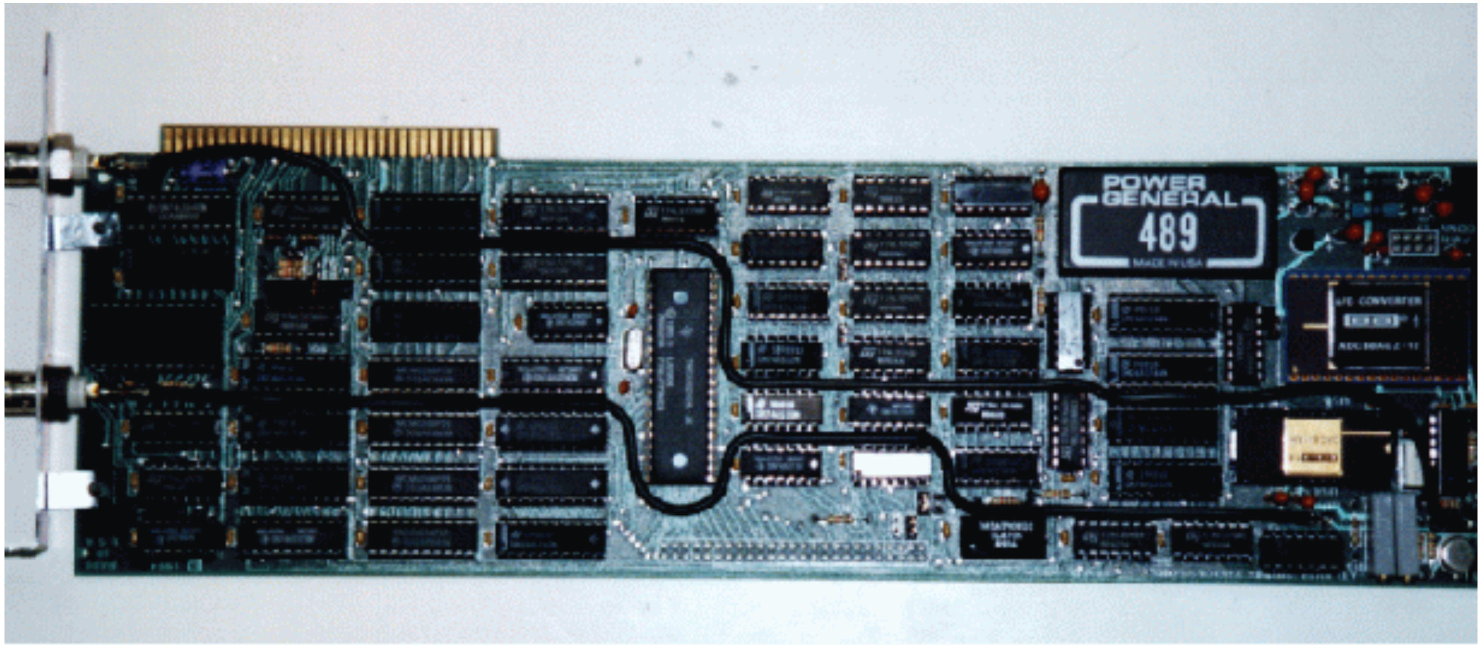
- INTRODUCE FILTERING IDEA
  - Weighted Average
  - Running Average
- FINITE IMPULSE RESPONSE FILTERS
  - **FIR** Filters
  - Show how to **compute** the output y[n] from the input signal, x[n]

Digital Filtering



- CONCENTRATE on the COMPUTER
  - PROCESSING ALGORITHMS
  - SOFTWARE (MATLAB)
  - HARDWARE: DSP chips, VLSI
- **DSP:** DIGITAL SIGNAL PROCESSING

The TMS32010, 1983

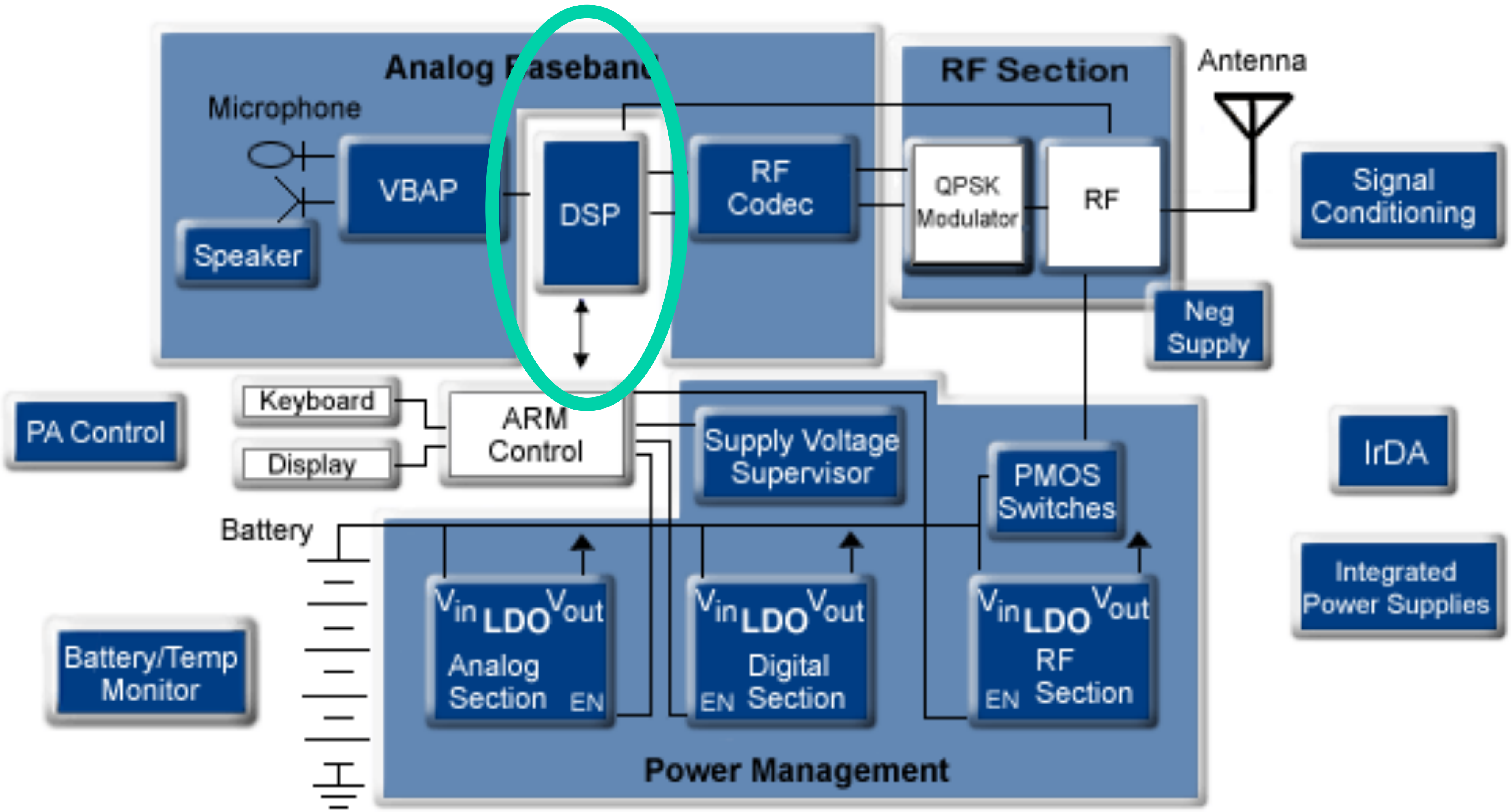


First PC plug-in board from Atlanta Signal Processors Inc.

Rockland Digital Filter, 1971

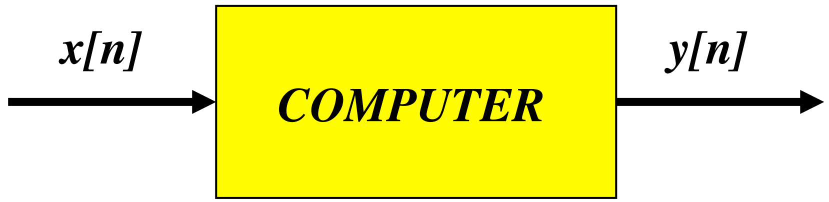
For the price of a small house, you could have one of these.

Digital Cell Phone (circa. 2000)



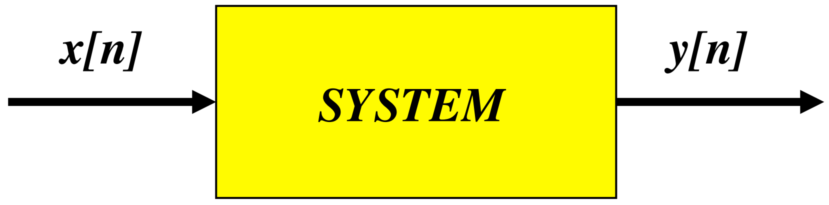
Now it plays video

DISCRETE-TIME SYSTEM



- OPERATE on x[n] to get y[n]
- WANT a **GENERAL** CLASS of SYSTEMS
  - **ANALYZE** the SYSTEM
    - TOOLS: TIME-DOMAIN & FREQUENCY-DOMAIN
  - **SYNTHESIZE** the SYSTEM

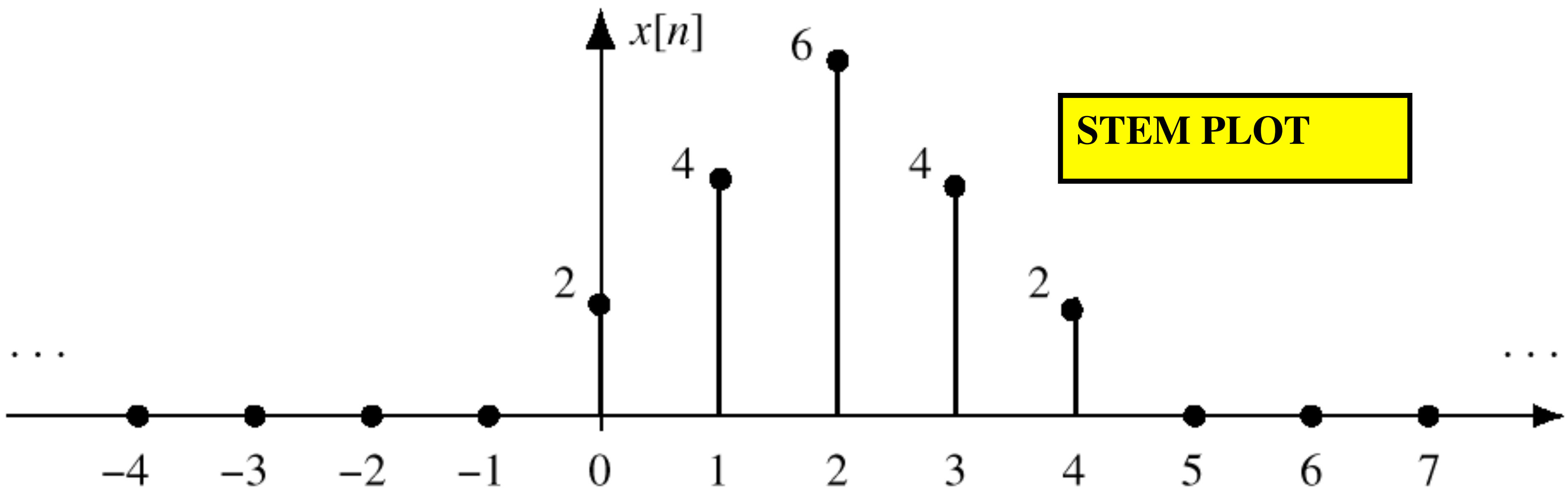
D-T System Examples



- EXAMPLES:
  - POINTWISE OPERATORS
    - SQUARING: $y[n] = (x[n])^2$
  - RUNNING AVERAGE
    - RULE: "the output at time n is the average of three consecutive input values"

Discrete-Time Signal

- x[n] is a LIST of NUMBERS
  - INDEXED by “n”



3-PT Average System

- ADD 3 CONSECUTIVE NUMBERS
  - Do this for each "n"

the following input–output equation

Make a TABLE

$$y[n] = \tfrac{1}{3}(x[n] + x[n+1] + x[n+2])$$

n	$n < -2$	-2	-1	0	1	2	3	4	5	$n > 5$
$x[n]$	0	0	0	2	4	**6**	**4**	**2**	0	0
$y[n]$	0	$\frac{2}{3}$	2	4	$\frac{14}{3}$	**4**	2	$\frac{2}{3}$	0	0

***n*=0** $y[0] = \frac{1}{3}(x[0] + x[1] + x[2])$

***n*=1** $y[1] = \frac{1}{3}(x[1] + x[2] + x[3])$

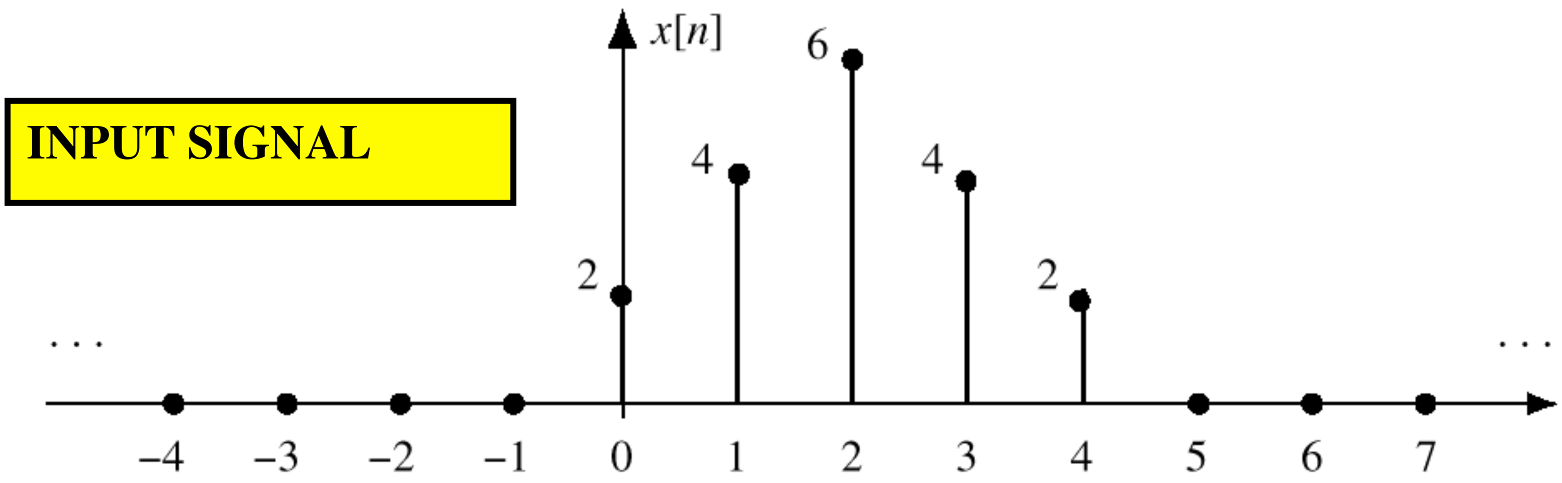


Figure 5.2 Finite-length input signal, $x[n]$.

$$y[n] = \tfrac{1}{3}(x[n] + x[n+1] + x[n+2])$$

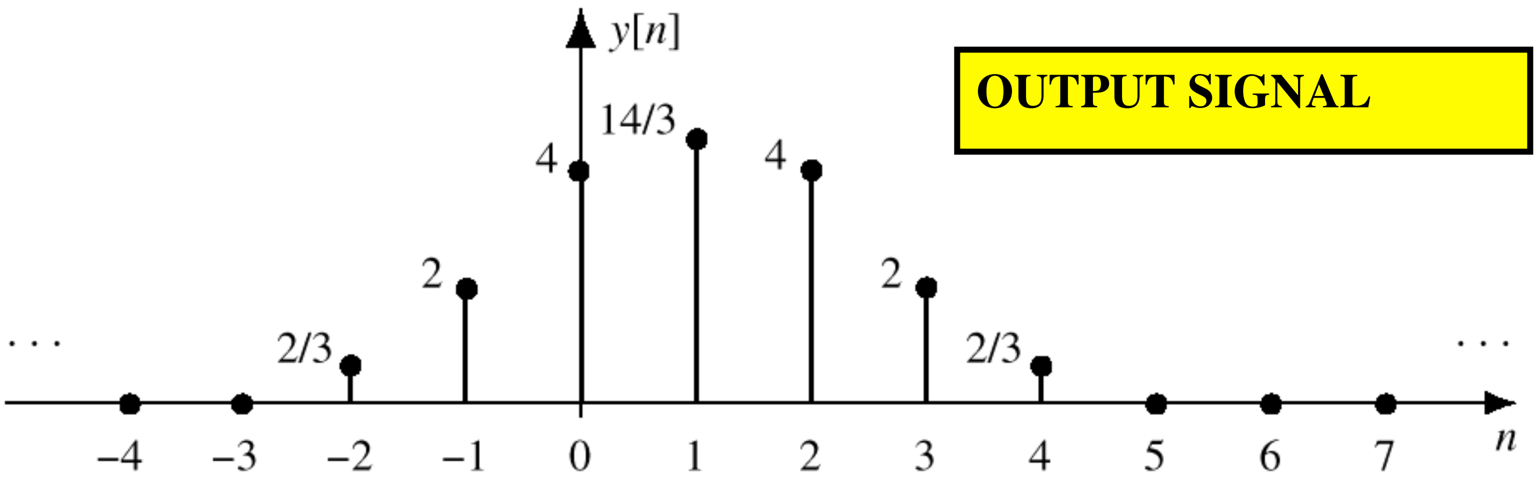


Figure 5.3 Output of running average, $y[n]$.

Past, Present, and Future

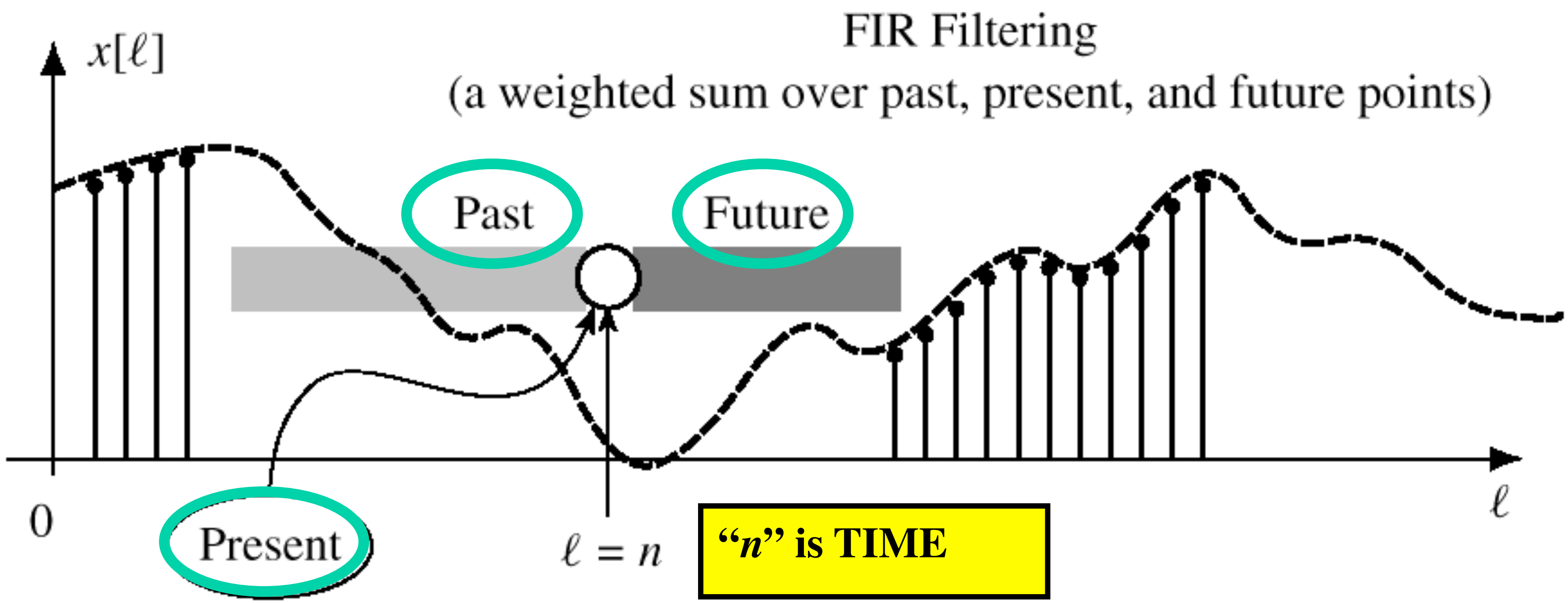


Figure 5.4 The running-average filter calculation at time index n uses values within a sliding window (shaded). Dark shading indicates the future ($\ell > n$); light shading, the past ($\ell < n$).

Another 3-pt Averager

- Uses “PAST” VALUES of x[n]
  - IMPORTANT IF “n” represents REAL TIME
    - WHEN x[n] & y[n] ARE STREAMS

$$y[n] = \tfrac{1}{3}(x[n] + x[n-1] + x[n-2])$$

n	$n < -2$	-2	-1	0	1	2	3	4	5	6	7	$n > 7$
$x[n]$	0	0	0	2	4	**6**	**4**	**2**	0	0	0	0
$y[n]$	0	0	0	$\frac{2}{3}$	2	4	$\frac{14}{3}$	**4**	2	$\frac{2}{3}$	0	0

General FIR Filter

- FILTER COEFFICIENTS $\{b_k\}$
  - DEFINE THE FILTER

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

  - For example,

$$\boxed{b_k = \{3, -1, 2, 1\}}$$

$$y[n] = \sum_{k=0}^{3} b_k x[n-k]$$

$$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

General FIR Filter

- FILTER COEFFICIENTS $\{b_k\}$

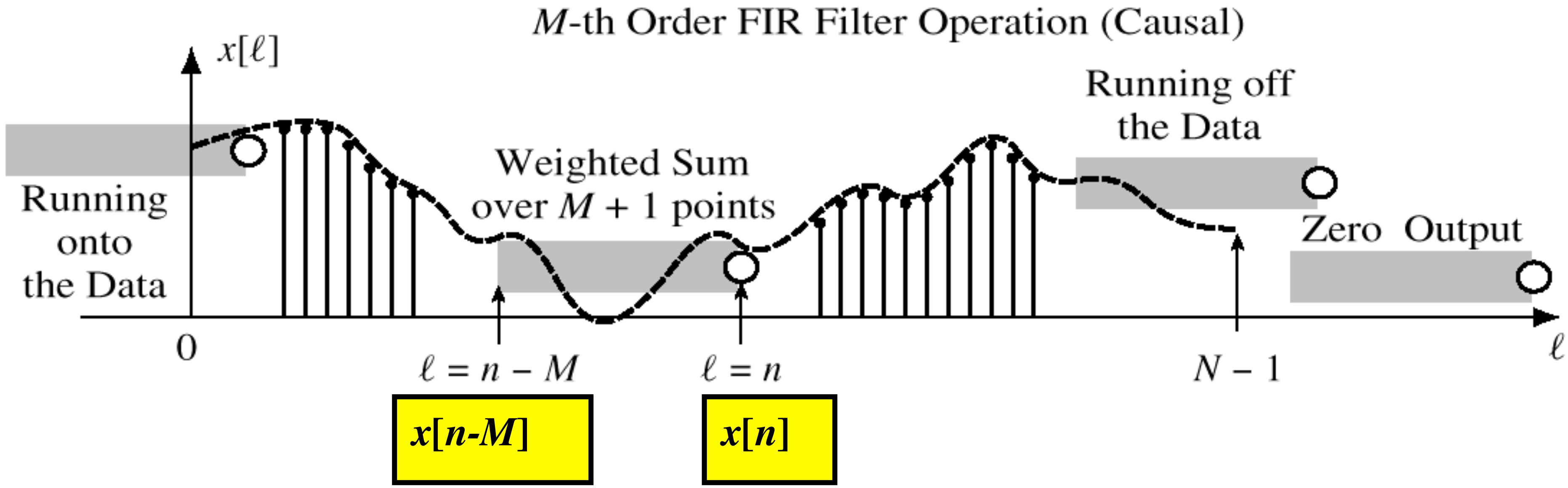
$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

- FILTER **ORDER** is M
- FILTER **LENGTH** is L = M+1
  - NUMBER of FILTER COEFFS is L

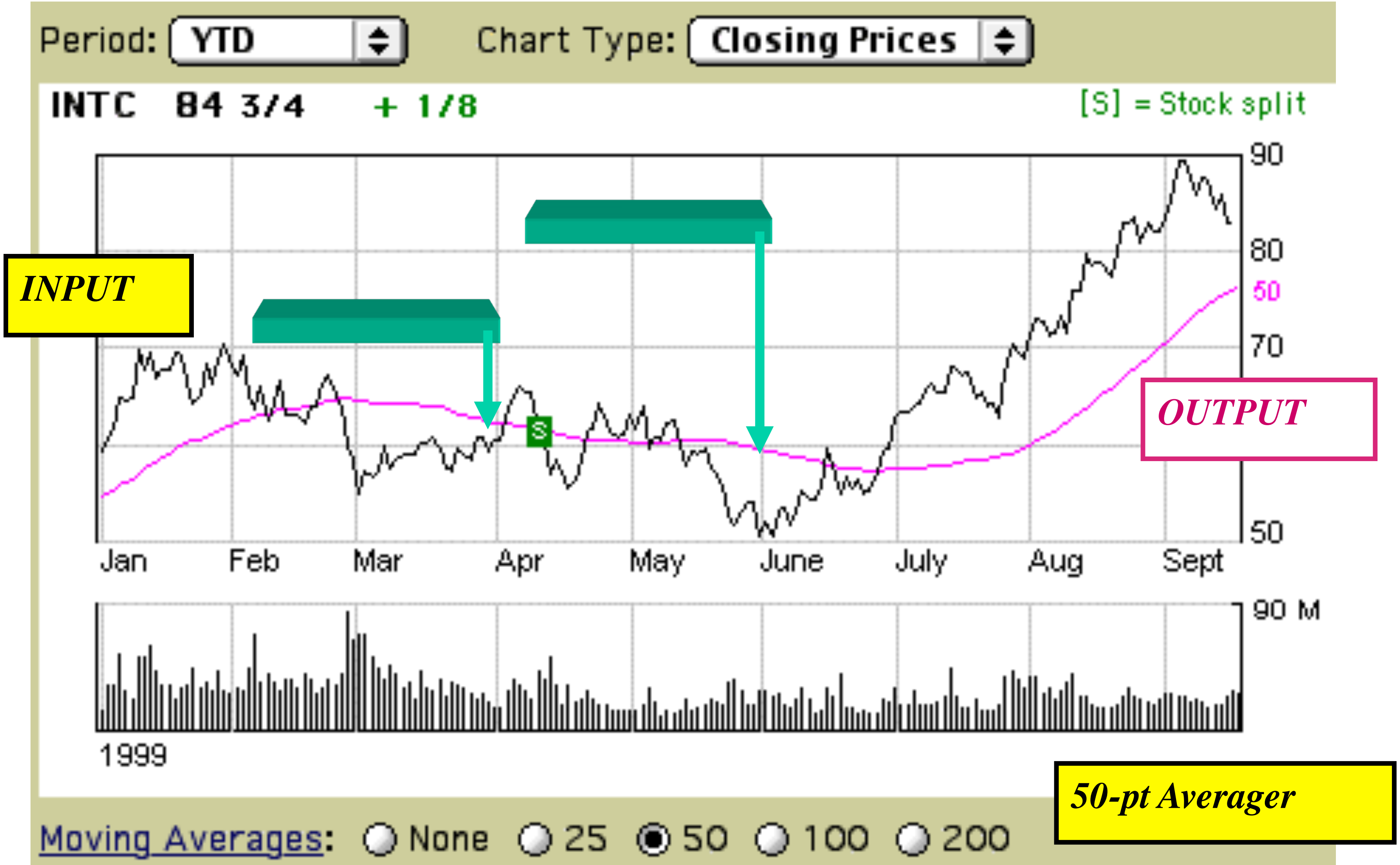
General FIR Filter

- SLIDE a WINDOW across x[n]

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$



Filtered Stock Signal



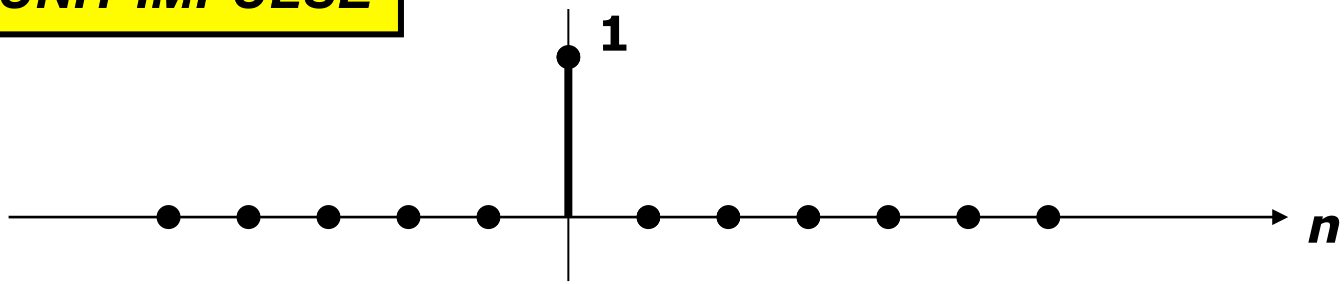
Special Input Signals

- x[n] = SINUSOID
- x[n] has only one NON-ZERO VALUE

FREQUENCY RESPONSE (LATER)

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

UNIT-IMPULSE



UNIT IMPULSE SIGNAL **δ[n]**

n	...	-2	-1	0	1	2	3	4	5	6	...
$\delta[n]$	0	0	0	$\boxed{1}$	0	0	0	0	0	0	0
$\delta[n-3]$	0	0	0	0	0	0	$\boxed{1}$	0	0	0	0

$n = 3$

δ[n] is NON-ZERO When its argument is equal to ZERO

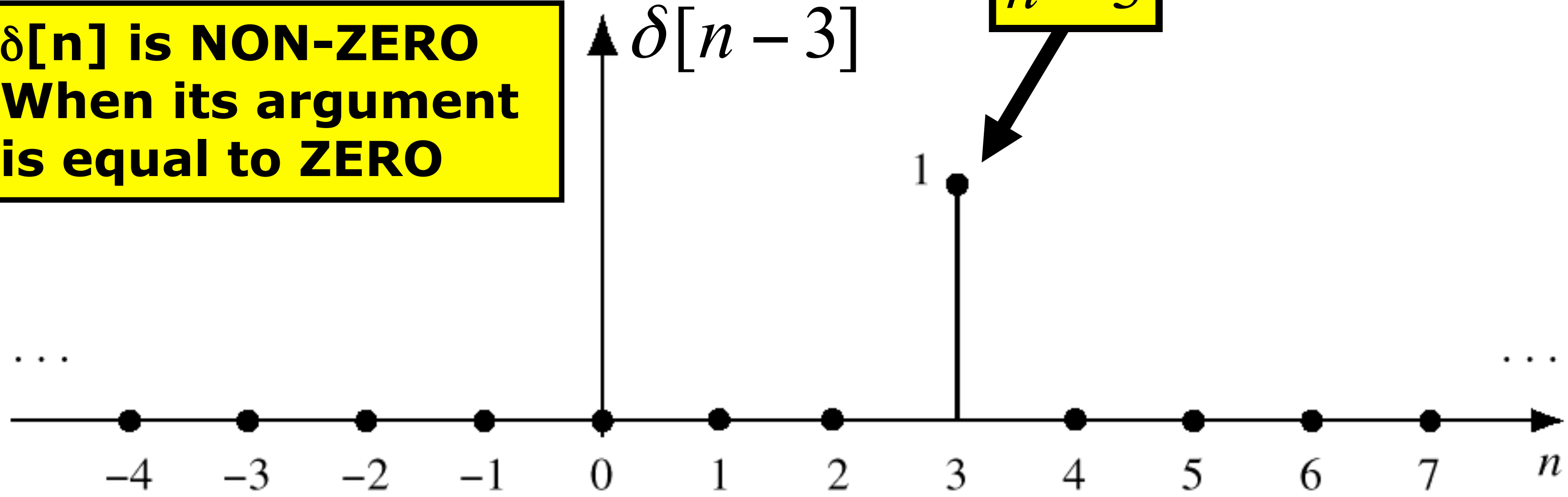
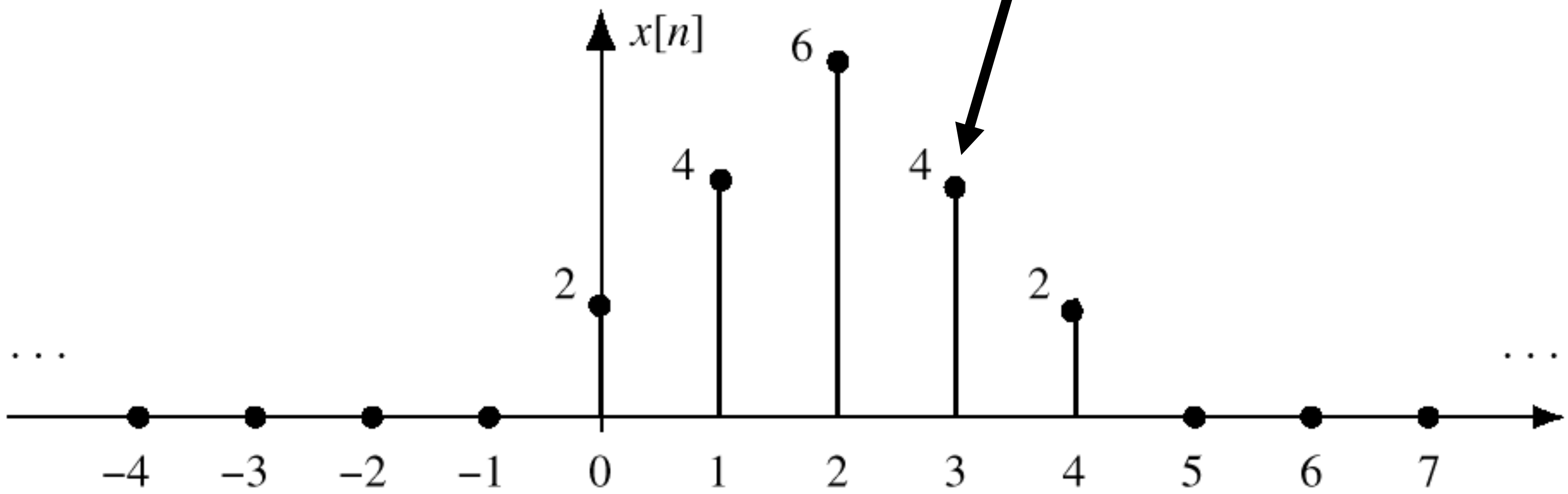


Figure 5.7 Shifted impulse sequence, $\delta[n-3]$.

MATH FORMULA for x[n]

- Use SHIFTED IMPULSES to write x[n]

$$x[n] = 2\delta[n] + 4\delta[n-1] + 6\delta[n-2] + 4\delta[n-3] + 2\delta[n-4]$$



Sum of Shifted Impulses

n	$\dots$	-2	-1	0	1	2	3	4	5	6	$\dots$
$2\delta[n]$	0	0	0	$\boxed{2}$	0	0	0	0	0	0	0
$4\delta[n-1]$	0	0	0	0	$\boxed{4}$	0	0	0	0	0	0
$6\delta[n-2]$	0	0	0	0	0	$\boxed{6}$	0	0	0	0	0
$4\delta[n-3]$	0	0	0	0	0	0	$\boxed{4}$	0	0	0	0
$2\delta[n-4]$	0	0	0	0	0	0	0	$\boxed{2}$	0	0	0
$x[n]$	0	0	0	2	4	6	4	2	0	0	0

$$x[n] = \sum_k x[k]\delta[n-k]$$

This formula ALWAYS works

$$= \dots + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + \dots \quad (5.3.6)$$

4-pt Averager

- CAUSAL SYSTEM: USE PAST VALUES

$$y[n] = \tfrac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

- INPUT = UNIT IMPULSE SIGNAL = **δ[n]**

$$x[n] = \delta[n]$$

$$y[n] = \tfrac{1}{4}\delta[n] + \tfrac{1}{4}\delta[n-1] + \tfrac{1}{4}\delta[n-2] + \tfrac{1}{4}\delta[n-3]$$

- OUTPUT is called "IMPULSE RESPONSE"

$$h[n] = \{\ldots, 0, 0, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}, 0, 0, \ldots\}$$

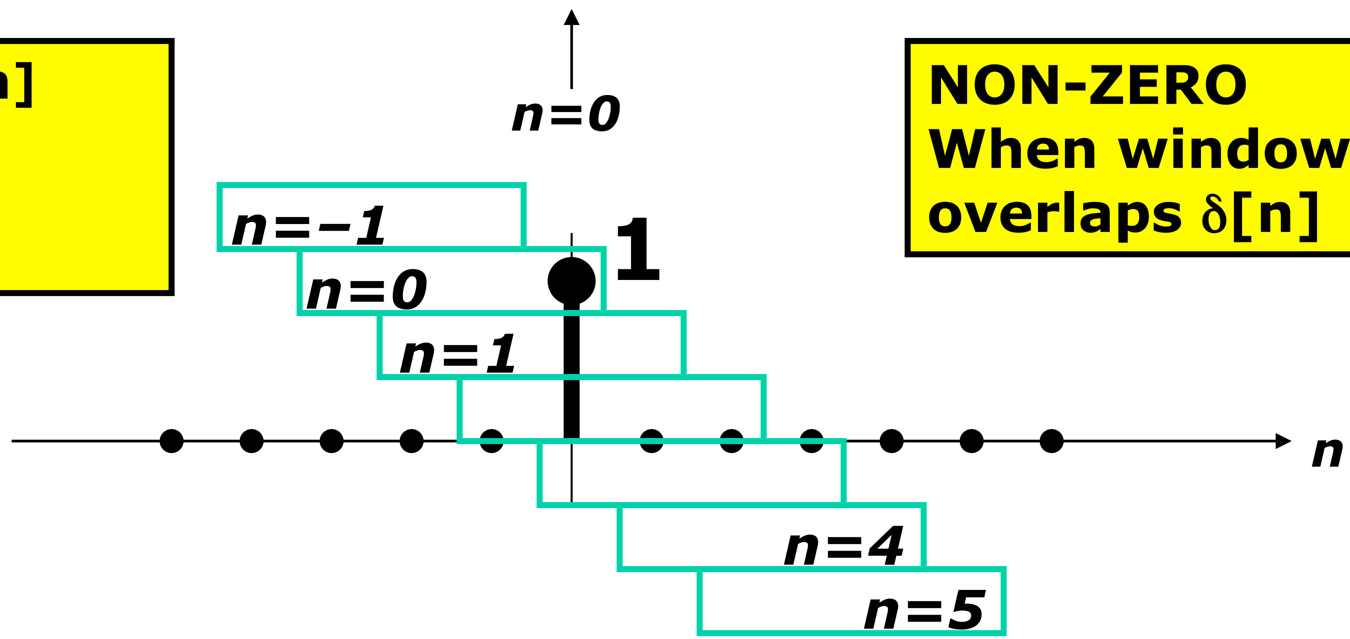
4-pt Avg Impulse Response

$$y[n] = \tfrac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

δ[n] "READS OUT" the FILTER COEFFICIENTS

$$h[n] = \{\ldots, 0, 0, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}, 0, 0, \ldots\}$$

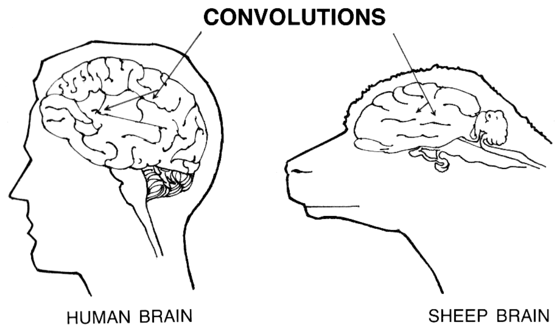
"h" in *h*[n] denotes Impulse Response



NON-ZERO When window overlaps δ[n]

FIR Impulse Response

- Convolution = Filter Definition
  - Filter Coeffs = Impulse Response



n	$n<0$	0	1	2	3	$\dots$	M	$M+1$	$n>M+1$
$x[n]=\delta[n]$	0	1	0	0	0	0	0	0	0
$y[n]=h[n]$	0	b_0	b_1	b_2	b_3	$\dots$	b_M	0	0

$$y[n]=\sum_{k=0}^{M} b_k x[n-k]$$

$$y[n]=\sum_{k=0}^{M} h[k]x[n-k]$$

CONVOLUTION

Filtering Example

- 7-point AVERAGER
  - Removes cosine
    - By making its amplitude (A) smaller

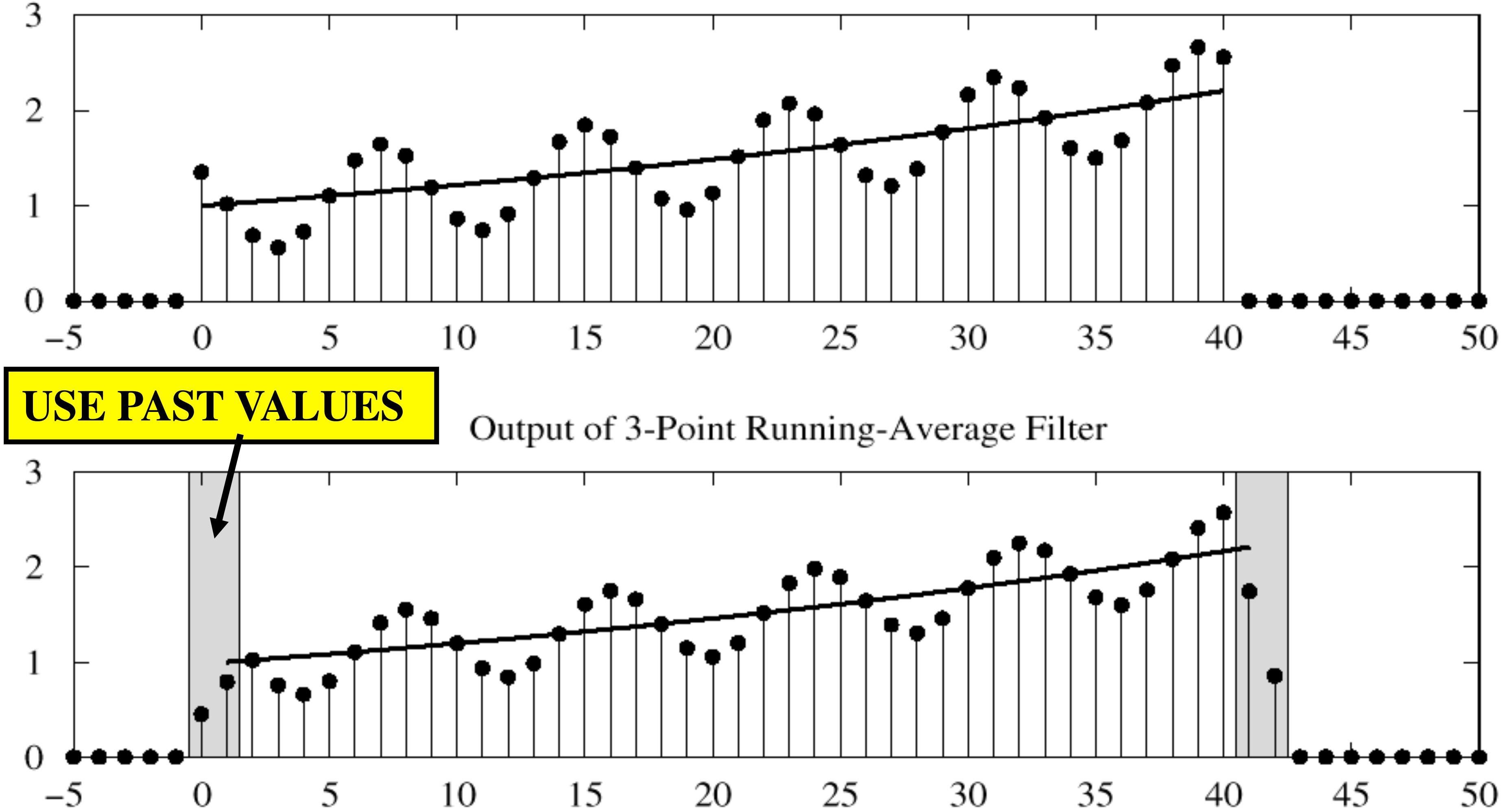
$$y_7[n] = \sum_{k=0}^{6} \left(\tfrac{1}{7}\right) x[n-k]$$

- 3-point AVERAGER
  - Changes A slightly

$$y_3[n] = \sum_{k=0}^{2} \left(\tfrac{1}{3}\right) x[n-k]$$

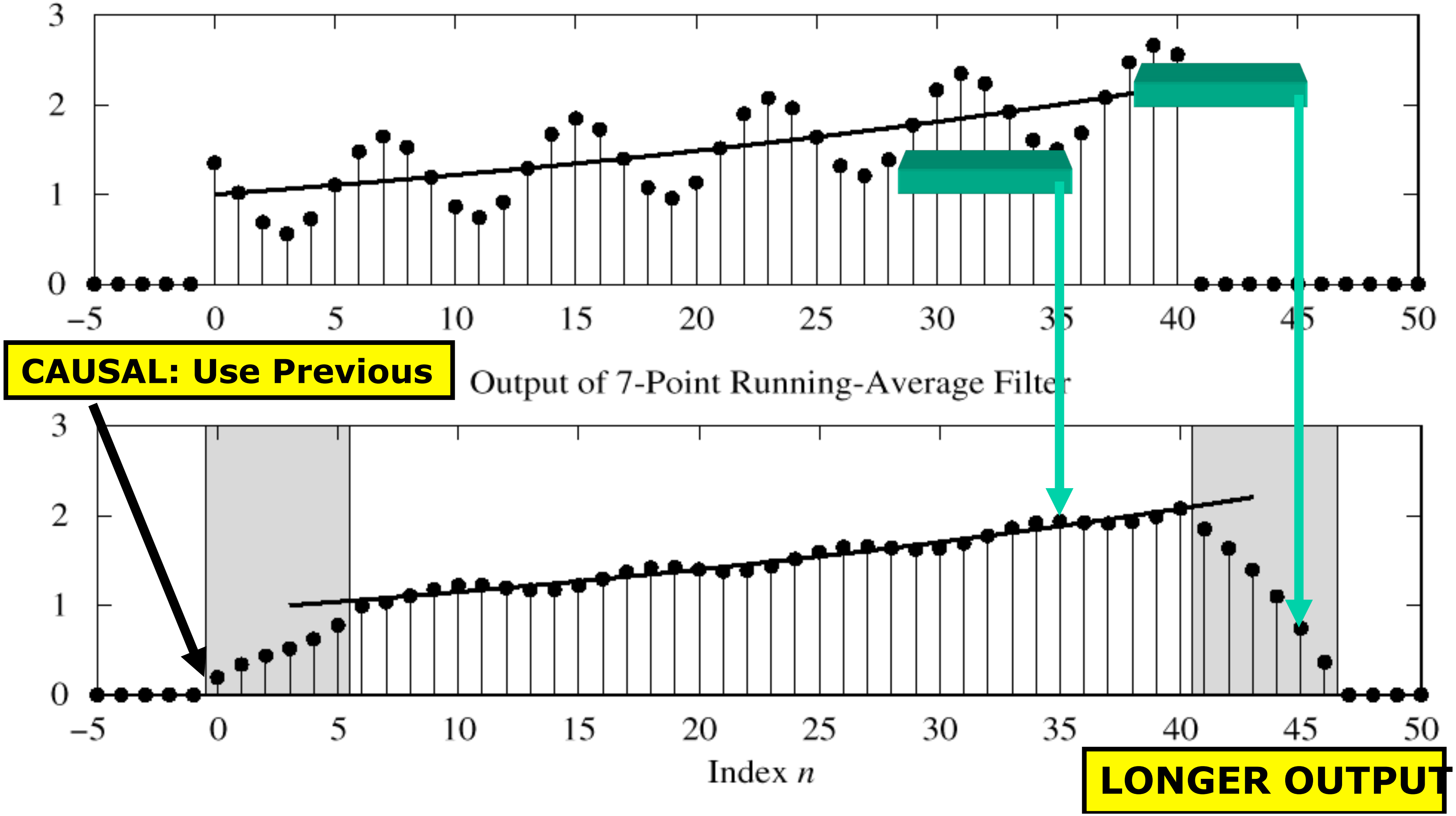
3-pt AVG EXAMPLE

Input : $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad$ for $0 \le n \le 40$



7-pt FIR EXAMPLE (AVG)

$$\text{Input}: x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$$



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