

# *Circuits and Systems I*

LECTURE #12

Frequency Response of FIR

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# Outline - Today

- Today <> Section 6-1  
Section 6-2  
Section 6-3  
Section 6-4
- Next week <> Section 6-6  
Section 6-7  
Section 6-8

**CSI**  
Progress  
Level:



# Lecture Objectives

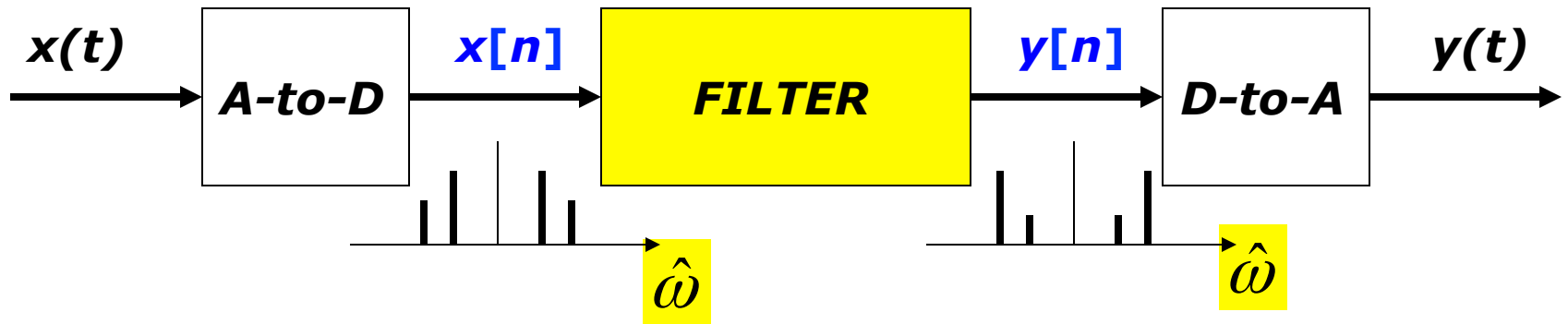
- **SINUSOIDAL** INPUT SIGNAL
  - DETERMINE the FIR FILTER OUTPUT
- **FREQUENCY RESPONSE** of FIR
  - PLOTTING vs. Frequency
  - MAGNITUDE vs. Freq
  - PHASE vs. Freq

$$H(e^{j\hat{\omega}}) = |H(e^{j\hat{\omega}})| e^{j\angle H(e^{j\hat{\omega}})}$$

# DOMAINS: Time & Frequency

- **Time-Domain: “n” = time**
  - $x[n]$  discrete-time signal
  - $x(t)$  continuous-time signal
- **Frequency Domain (sum of sinusoids)**
  - Spectrum vs.  $f$  (Hz)
    - ANALOG vs. DIGITAL
  - Spectrum vs.  $\omega$ -hat
- Move back and forth QUICKLY

# Digital "Filtering"



- CONCENTRATE on the **SPECTRUM**
- SINUSOIDAL INPUT
  - INPUT  $x[n] = \text{SUM of SINUSOIDS}$
  - Then, OUTPUT  $y[n] = \text{SUM of SINUSOIDS}$

# Filtering Example

- 7-point AVERAGER

- Removes cosine

- By making its amplitude (A) smaller

$$y_7[n] = \sum_{k=0}^6 \left(\frac{1}{7}\right) x[n-k]$$

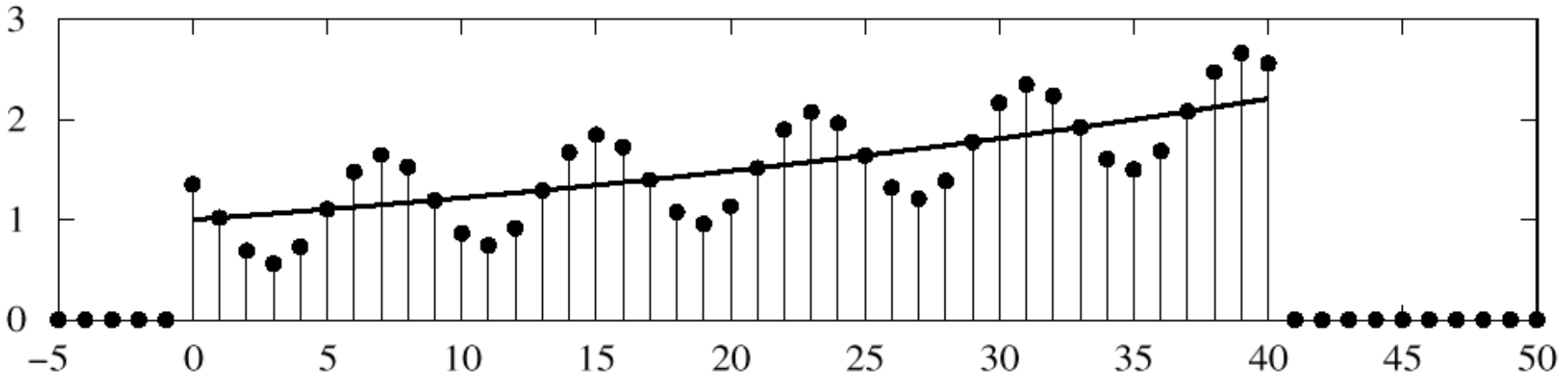
- 3-point AVERAGER

- Changes A slightly

$$y_3[n] = \sum_{k=0}^2 \left(\frac{1}{3}\right) x[n-k]$$

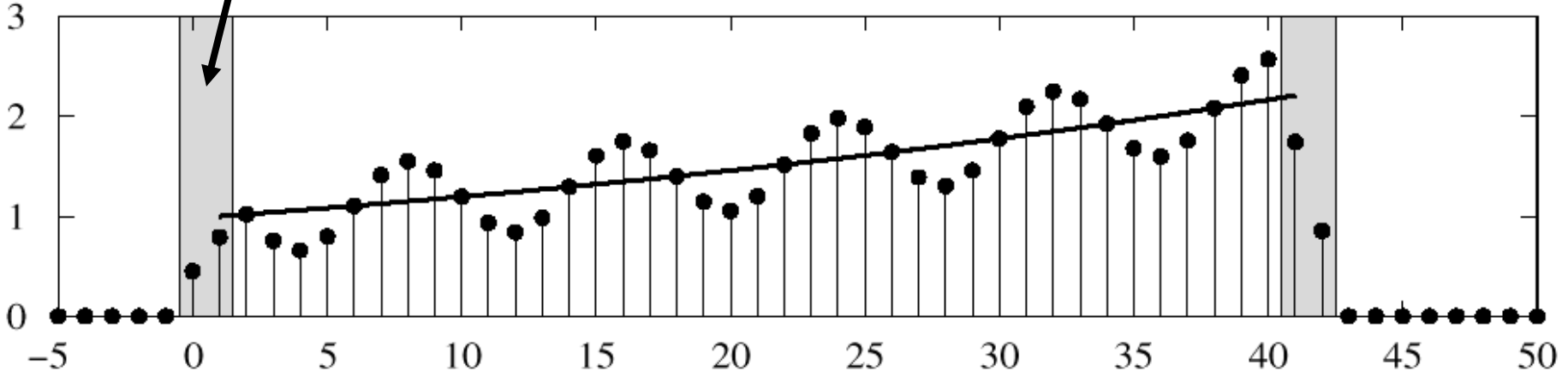
# 3-pt Averager Example

Input :  $x[n] = (1.02)^n + \cos(2\pi n / 8 + \pi / 4)$  for  $0 \leq n \leq 40$



**USE PAST VALUES**

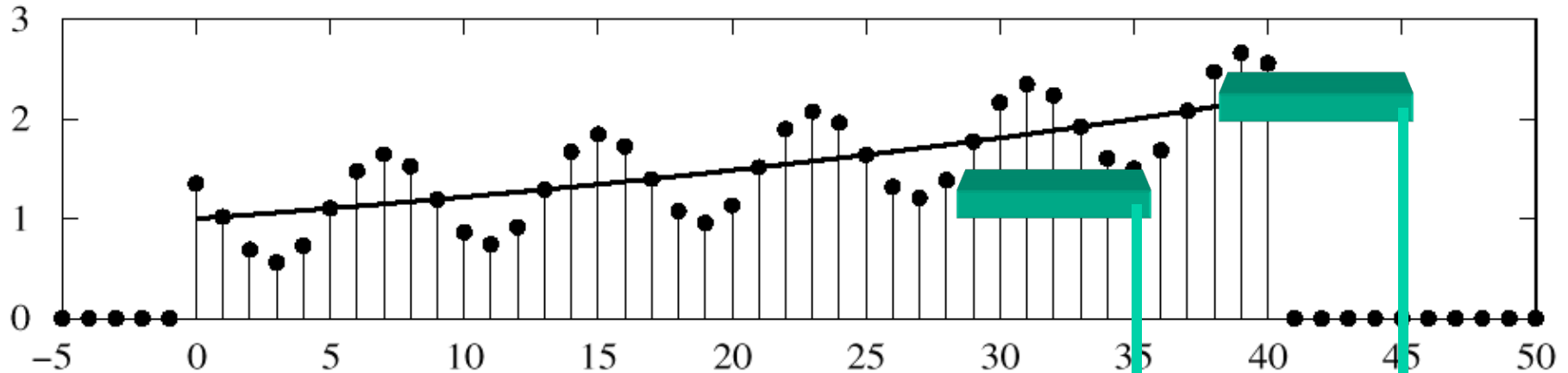
Output of 3-Point Running-Average Filter





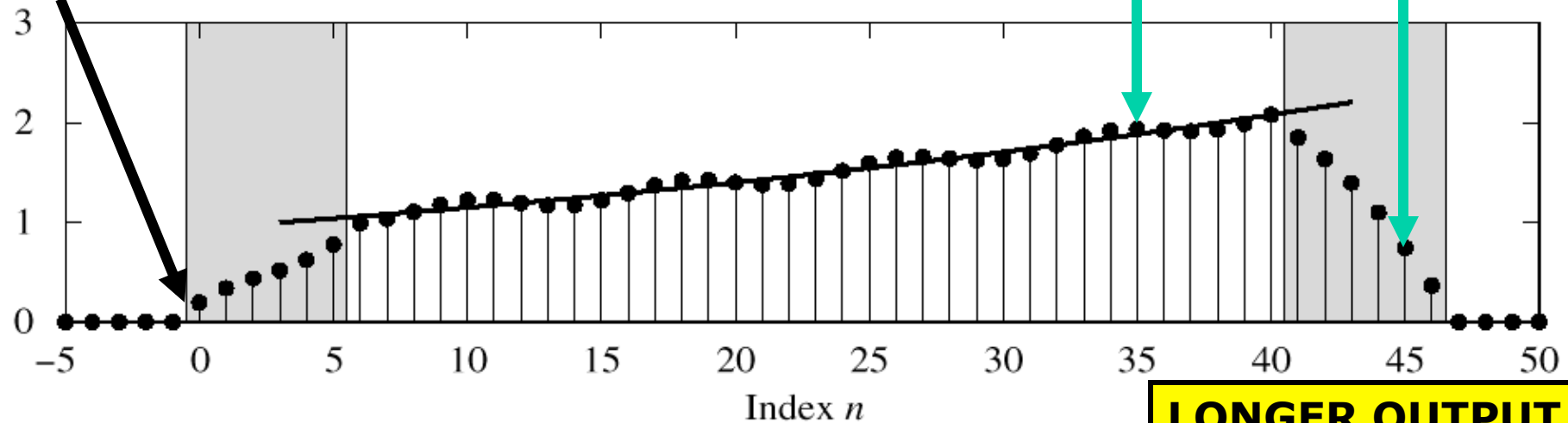
# 7-pt FIR Example (FIR)

Input :  $x[n] = (1.02)^n + \cos(2\pi n / 8 + \pi / 4)$  for  $0 \leq n \leq 40$



**CAUSAL: Use Previous**

Output of 7-Point Running-Average Filter



**LONGER OUTPUT**

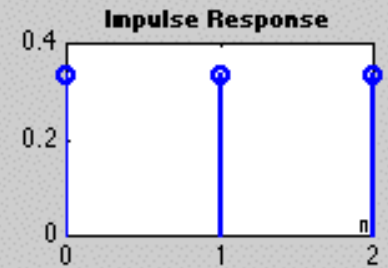
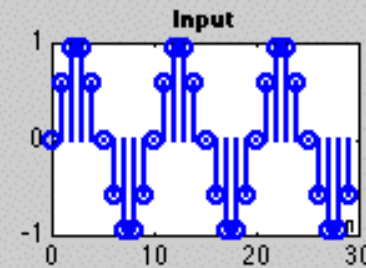
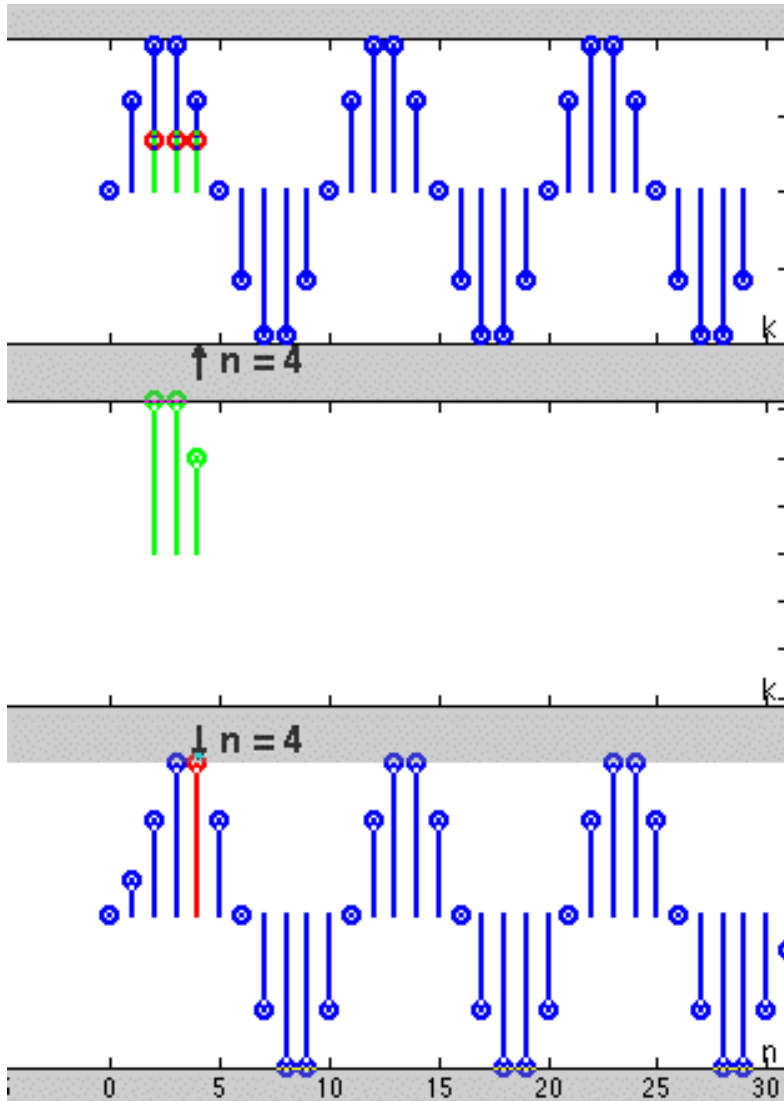
# Sinusoidal Response

- INPUT:  $x[n] = \text{SINUSOID}$
- OUTPUT:  $y[n]$  will also be a SINUSOID
  - Different Amplitude and Phase

– **SAME** Frequency

- AMPLITUDE & PHASE CHANGE
  - Called the **FREQUENCY RESPONSE**

# DCONVDEMO: MATLAB GUI



Get  $x[n]$

Get  $h[n]$

Flip  $x[n]$

Flip  $h[n]$

**Signal Axis:**  
 $\circ = x[k]$   
 $\circ = h[n-k]$

**Multiplication Axis:**  
 $x[k]h[n-k]$

**Convolution Axis:**  
 $y[n] = \sum x[k]h[n-k]$

Close

Help

# Complex Exponential

$$x[n] = Ae^{j\varphi} e^{j\hat{\omega}n} \quad -\infty < n < \infty$$

**$x[n]$  is the input signal—a complex exponential**

$$y[n] = \sum_{k=0}^M b_k x[n-k] = \sum_{k=0}^M h[k] x[n-k]$$

**FIR DIFFERENCE EQUATION**

# Complex Exponential Output

- Use the FIR “Difference Equation”

$$y[n] = \sum_{k=0}^M b_k x[n-k] = \sum_{k=0}^M b_k A e^{j\varphi} e^{j\hat{\omega}(n-k)}$$

$$= \left( \sum_{k=0}^M b_k e^{j\hat{\omega}(-k)} \right) A e^{j\varphi} e^{j\hat{\omega}n}$$

$$= H(\hat{\omega}) A e^{j\varphi} e^{j\hat{\omega}n}$$

# Frequency Response

- At each frequency, we can **DEFINE**

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k}$$

**FREQUENCY RESPONSE**

- Complex-valued formula
  - Has **MAGNITUDE** vs. frequency
  - And **PHASE** vs. frequency

- Notation:

$H(e^{j\hat{\omega}})$  in place of  $H(\hat{\omega})$

# EXAMPLE 6.1

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k}$$

$$\{b_k\} = \{1, 2, 1\}$$

$$\begin{aligned} H(e^{j\hat{\omega}}) &= 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} \\ &= e^{-j\hat{\omega}} (e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}}) \\ &= e^{-j\hat{\omega}} (2 + 2\cos\hat{\omega}) \end{aligned}$$

**EXPLOIT  
SYMMETRY**

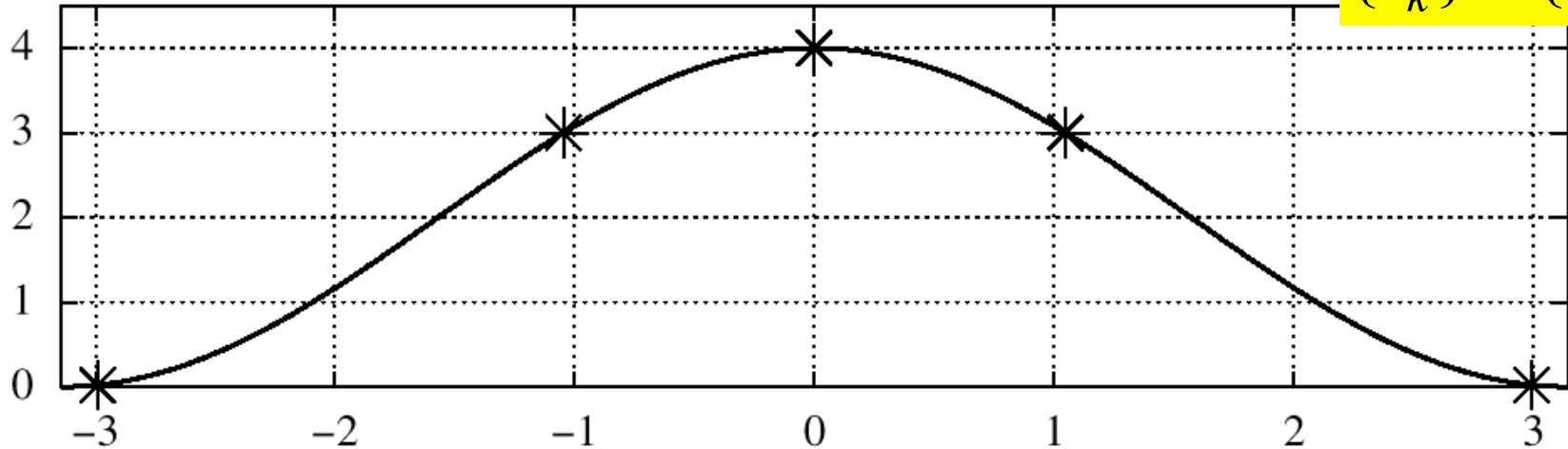
Since  $(2 + 2\cos\hat{\omega}) \geq 0$

Magnitude is  $|H(e^{j\hat{\omega}})| = (2 + 2\cos\hat{\omega})$

and Phase is  $\angle H(e^{j\hat{\omega}}) = -\hat{\omega}$

# PLOT of FREQ RESPONSE

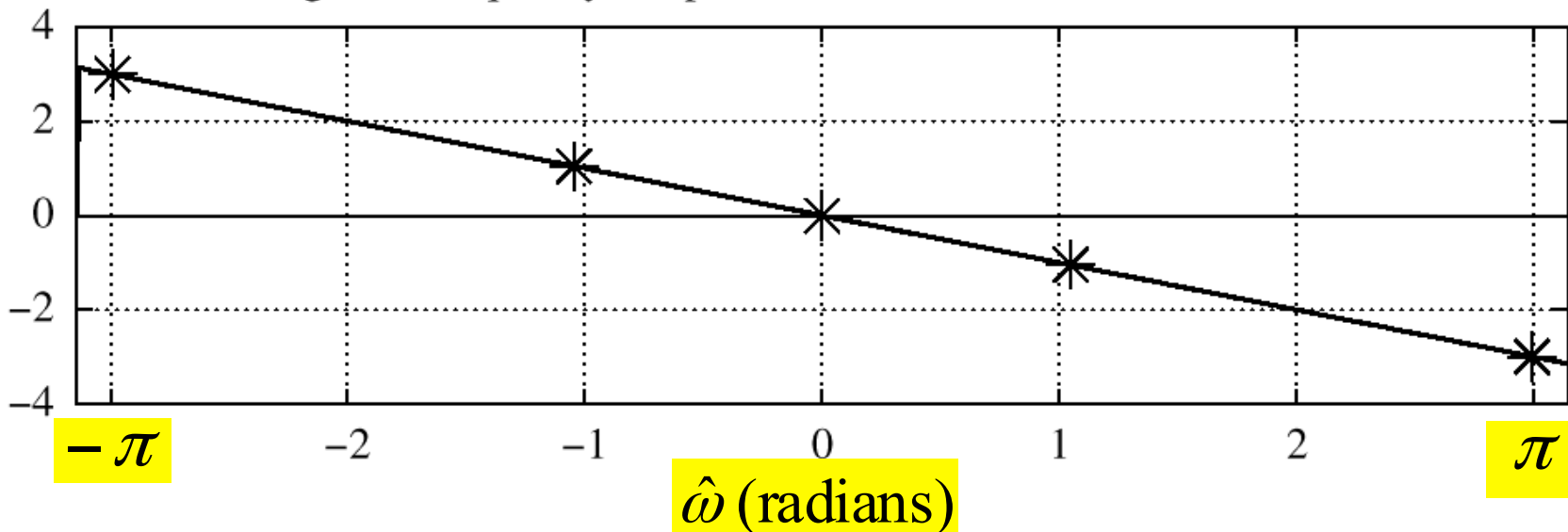
Magnitude of Frequency Response of FIR Filter with Coefficients  $\{b_k\} = \{1, 2, 1\}$



$\hat{\omega}$

$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

Phase Angle of Frequency Response of FIR Filter with Coefficients [1, 2, 1]



$-\pi$

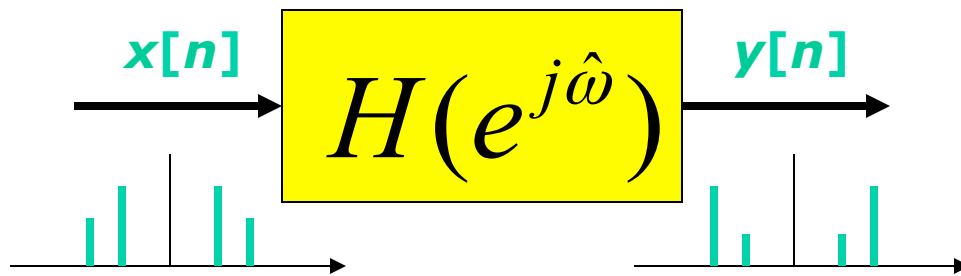
$\pi$

$\hat{\omega}$  (radians)



## EXAMPLE 6.2

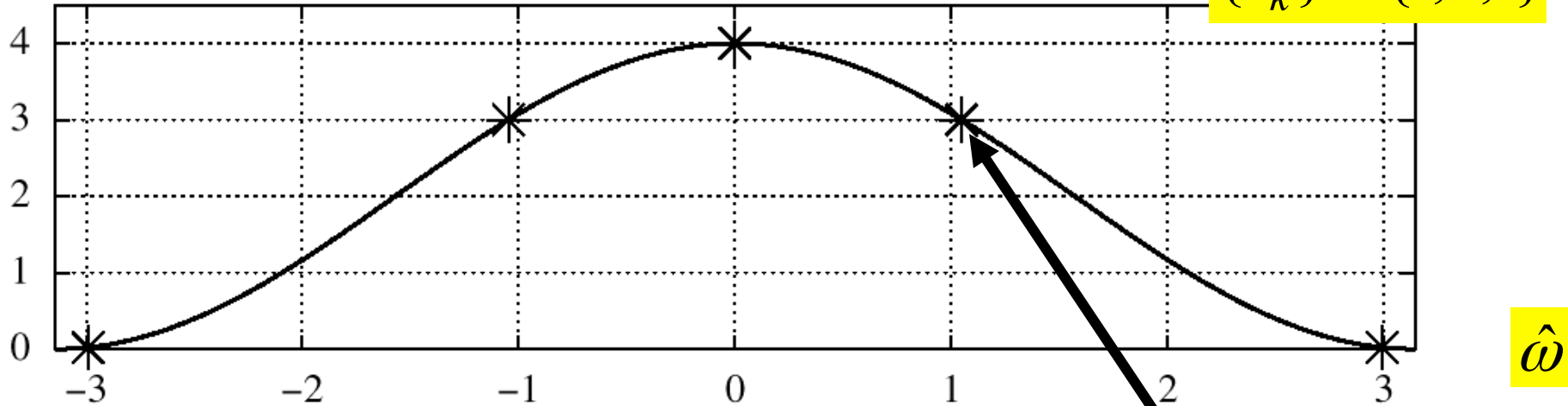
Find  $y[n]$  when  $H(e^{j\hat{\omega}})$  is known  
and  $x[n] = 2e^{j\pi/4} e^{j(\pi/3)n}$



$$H(e^{j\hat{\omega}}) = (2 + 2 \cos \hat{\omega}) e^{-j\hat{\omega}}$$

# PLOT of FREQ RESPONSE

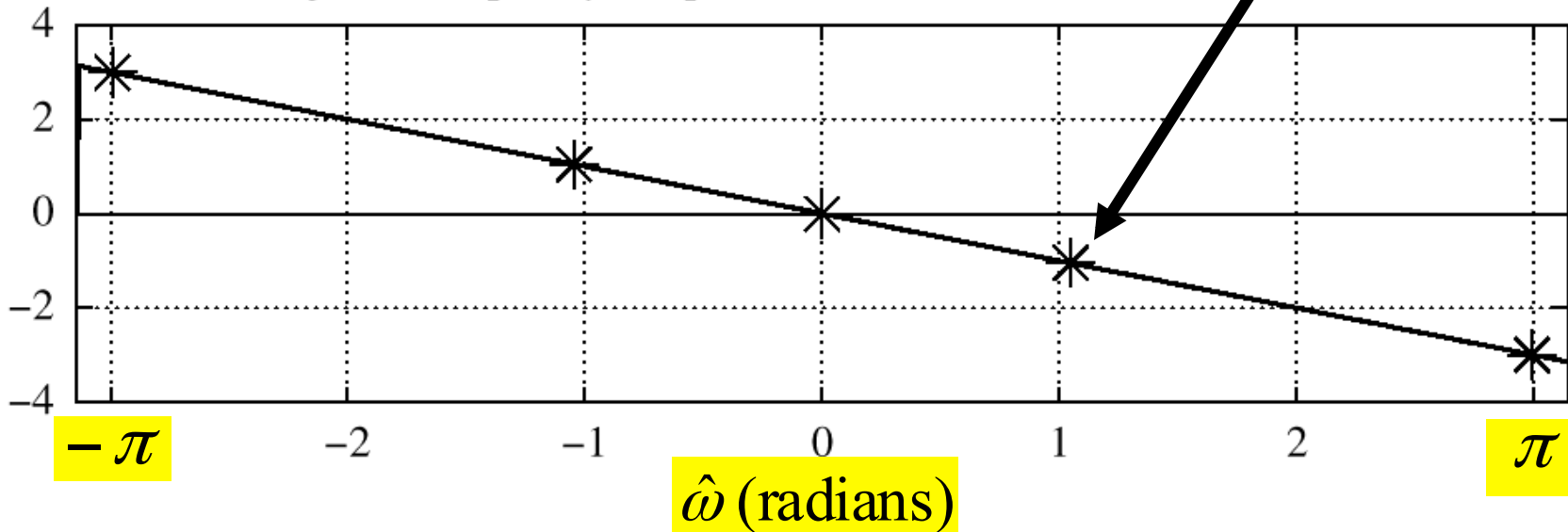
Magnitude of Frequency Response of FIR Filter with Coefficients  $\{b_k\} = \{1, 2, 1\}$



$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

**RESPONSE at  $\pi/3$**

Phase Angle of Frequency Response of FIR Filter with Coefficients [1, 2, 1]



## EXAMPLE 6.2 (answer)

Find  $y[n]$  when  $x[n] = 2e^{j\pi/4} e^{j(\pi/3)n}$

One Step - evaluate  $H(e^{j\hat{\omega}})$  at  $\hat{\omega} = \pi/3$

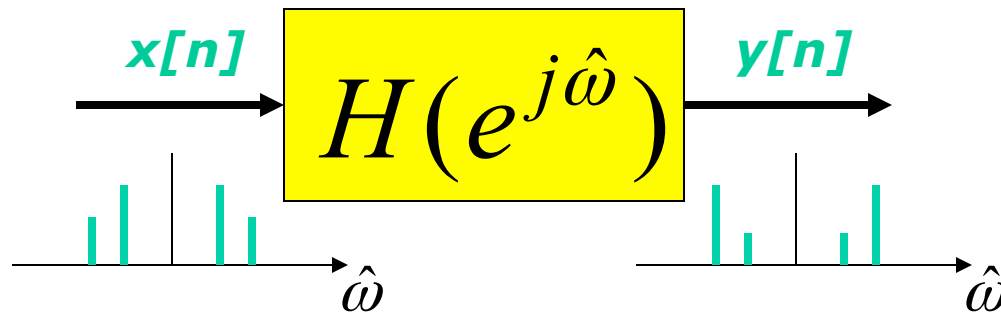
$$H(e^{j\hat{\omega}}) = (2 + 2 \cos \hat{\omega}) e^{-j\hat{\omega}}$$

$$H(e^{j\hat{\omega}}) = 3e^{-j\pi/3} \quad @ \hat{\omega} = \pi/3$$

$$y[n] = \left( 3e^{-j\pi/3} \right) \times 2e^{j\pi/4} e^{j(\pi/3)n} = 6e^{-j\pi/12} e^{j(\pi/3)n}$$

# Example: COSINE Input

Find  $y[n]$  when  $H(e^{j\hat{\omega}})$  is known  
and  $x[n] = 2 \cos(\frac{\pi}{3}n + \frac{\pi}{4})$



$$H(e^{j\hat{\omega}}) = (2 + 2 \cos \hat{\omega})e^{-j\hat{\omega}}$$

# EX: COSINE Input

Find  $y[n]$  when  $x[n] = 2 \cos\left(\frac{\pi}{3}n + \frac{\pi}{4}\right)$

$$2 \cos\left(\frac{\pi}{3}n + \frac{\pi}{4}\right) = e^{j(\pi n/3 + \pi/4)} + e^{-j(\pi n/3 + \pi/4)}$$

$$\Rightarrow x[n] = x_1[n] + x_2[n]$$

Use  
Linearity

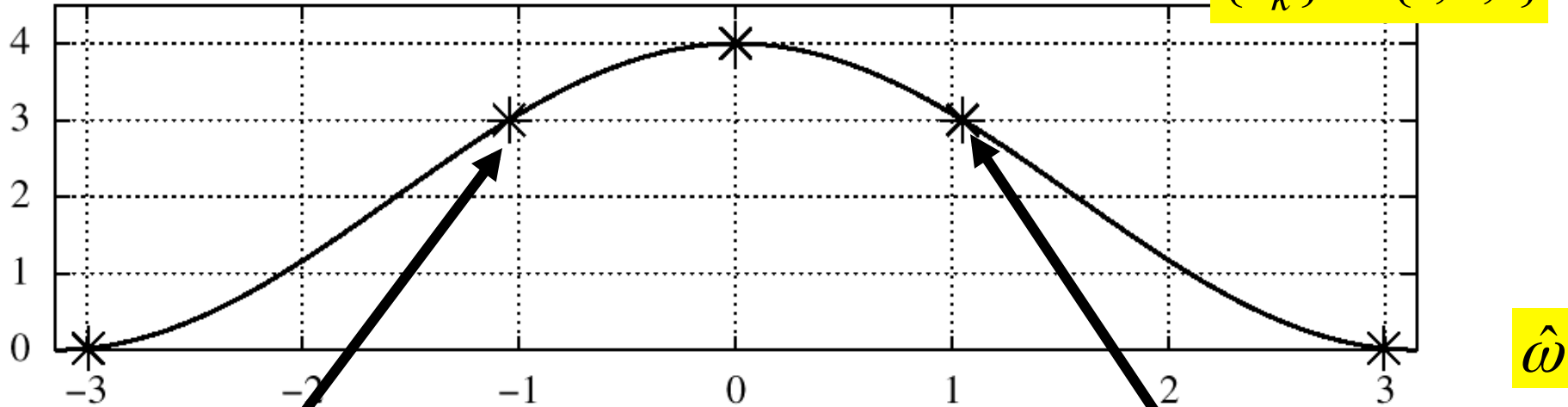
$$y_1[n] = H(e^{j\pi/3})e^{j(\pi n/3 + \pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3})e^{-j(\pi n/3 + \pi/4)}$$

$$\Rightarrow y[n] = y_1[n] + y_2[n]$$

# PLOT of FREQ RESPONSE

Magnitude of Frequency Response of FIR Filter with Coefficients  $\{b_k\} = \{1, 2, 1\}$

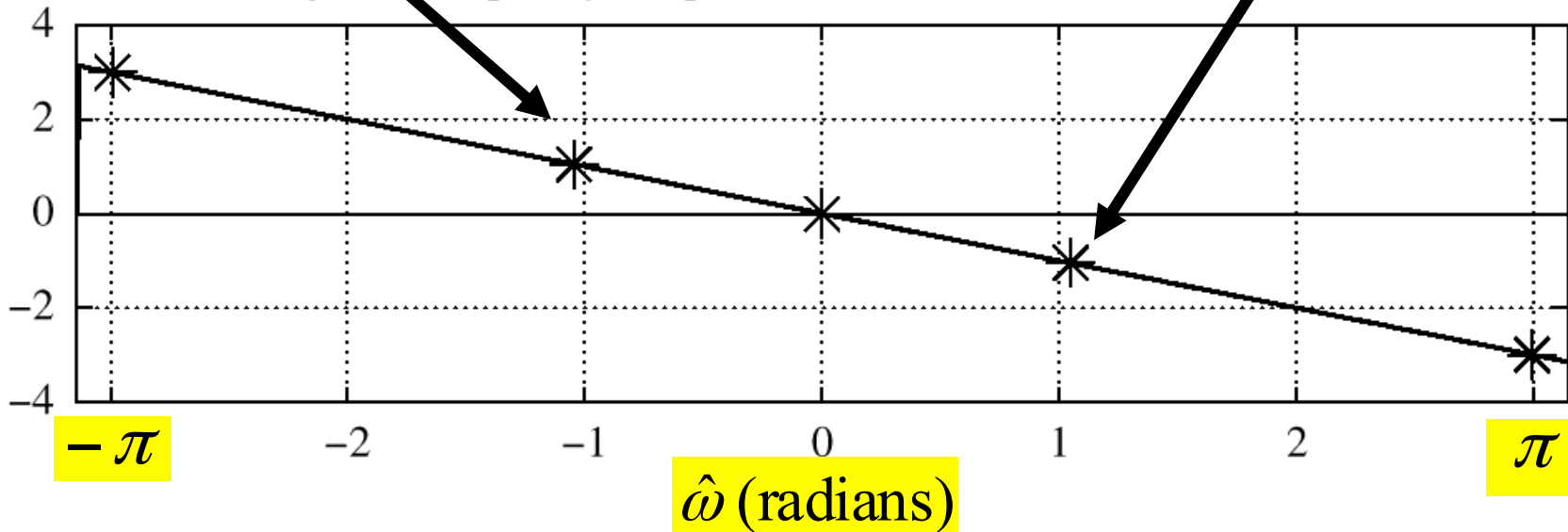


**RESPONSE at  $-\pi/3$**

$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

**RESPONSE at  $\pi/3$**

Phase Angle of Frequency Response of FIR Filter with Coefficients [1, 2, 1]



## EX: COSINE Input (ans-2)

Find  $y[n]$  when  $x[n] = 2 \cos\left(\frac{\pi}{3}n + \frac{\pi}{4}\right)$

$$H(e^{j\hat{\omega}}) = (2 + 2 \cos \hat{\omega})e^{-j\hat{\omega}}$$

$$y_1[n] = H(e^{j\pi/3})e^{j(\pi n/3 + \pi/4)} = 3e^{-j(\pi/3)}e^{j(\pi n/3 + \pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3})e^{-j(\pi n/3 + \pi/4)} = 3e^{j(\pi/3)}e^{-j(\pi n/3 + \pi/4)}$$

$$y[n] = 3e^{j(\pi n/3 - \pi/12)} + 3e^{-j(\pi n/3 - \pi/12)}$$

$$\Rightarrow y[n] = 6 \cos\left(\frac{\pi}{3}n - \frac{\pi}{12}\right)$$

# MATLAB: Frequency Response

- **HH = freqz(bb, 1, ww)**
  - VECTOR **bb** contains Filter Coefficients
  - SP-First: **HH = freekz(bb, 1, ww)**
- FILTER COEFFICIENTS  $\{b_k\}$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k}$$



# LTI Systems

- LTI: Linear & Time-Invariant
- COMPLETELY CHARACTERIZED by:
  - FREQUENCY RESPONSE, or
  - IMPULSE RESPONSE  $h[n]$
- Sinusoid IN -----> Sinusoid OUT
  - At the SAME Frequency

# Time & Frequency Relation

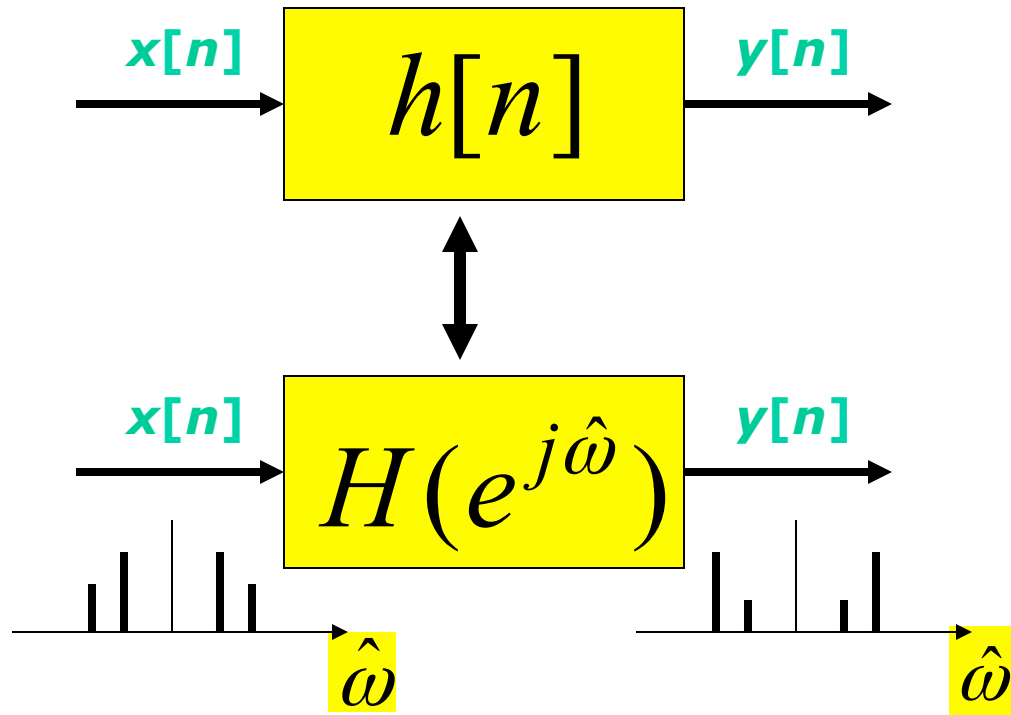
- Get Frequency Response from  $h[n]$ 
  - Here is the FIR case:

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k} = \sum_{k=0}^M h[k] e^{-j\hat{\omega}k}$$

**IMPULSE RESPONSE**

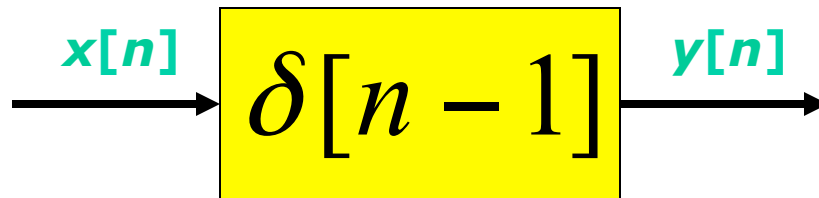
# Block Diagrams

- Equivalent Representations



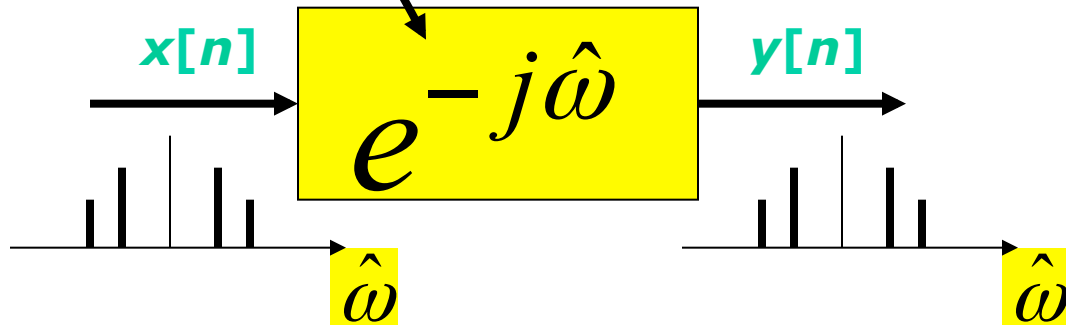
# Unit-Delay System

Find  $h[n]$  and  $H(e^{j\hat{\omega}})$  for  $y[n] = x[n - 1]$



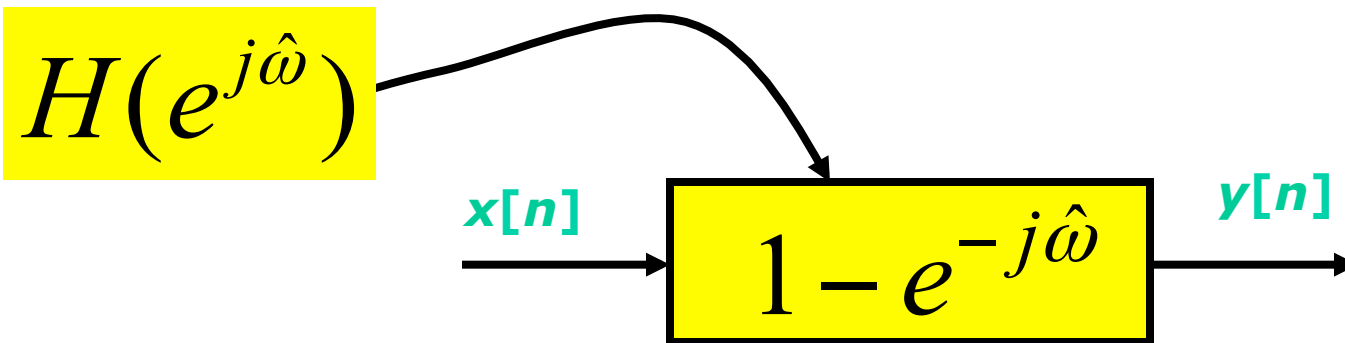
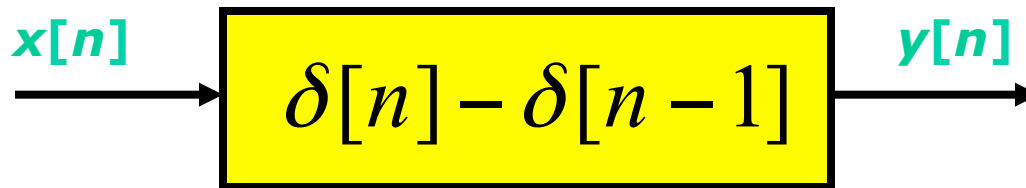
$$\{b_k\} = \{0, 1\}$$

$$H(e^{j\hat{\omega}})$$

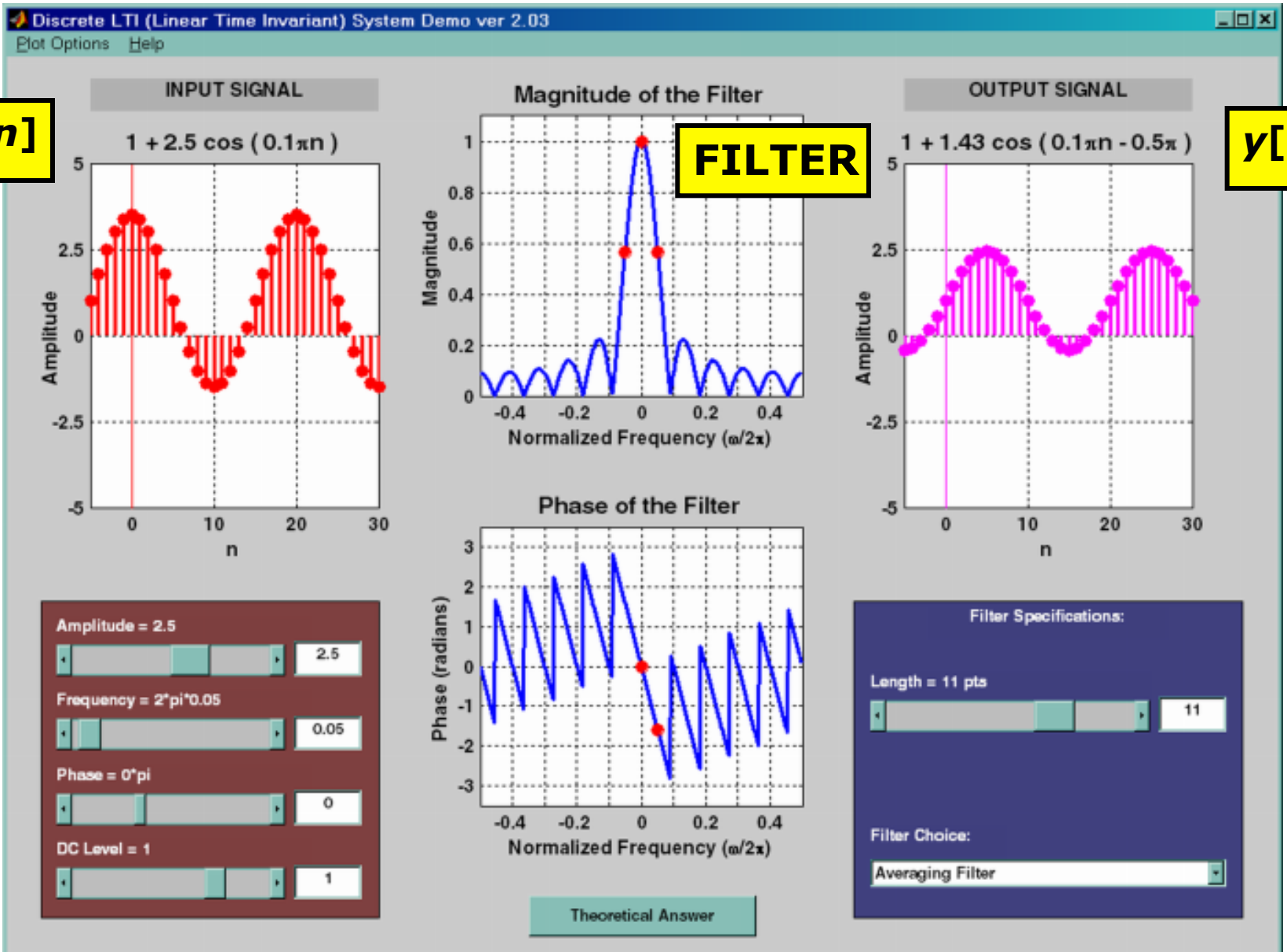


# First Difference System

Find  $h[n]$  and  $H(e^{j\hat{\omega}})$  for the Difference Equation:  $y[n] = x[n] - x[n - 1]$

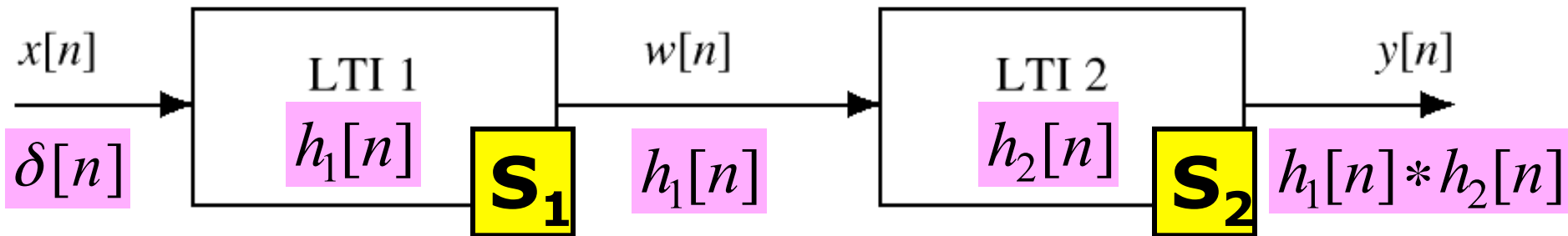


# DLTI Demo with Sinusoids



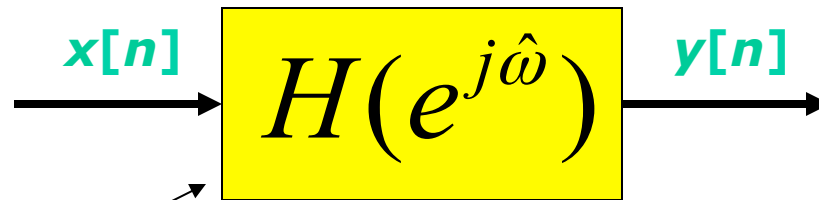
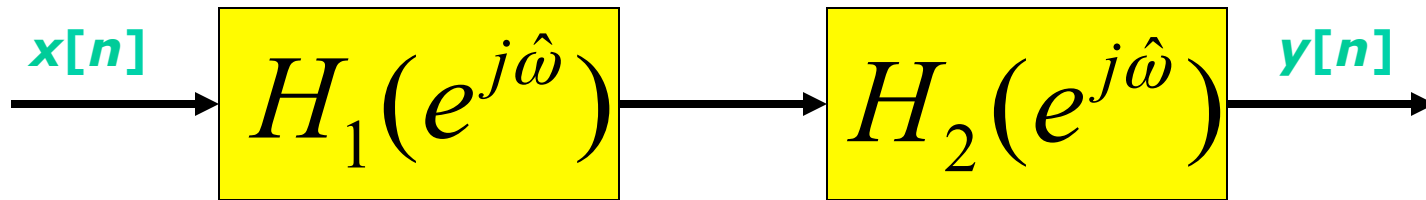
# Cascade Systems

- Does the order of  $S_1$  &  $S_2$  matter?
  - NO, **LTI SYSTEMS** can be rearranged !!!
  - WHAT ARE THE FILTER COEFFS?  $\{b_k\}$
  - WHAT is the overall FREQUENCY RESPONSE ?



# Cascade Equivalent

- MULTIPLY the Frequency Responses



**EQUIVALENT  
SYSTEM**

$$H(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}})H_2(e^{j\hat{\omega}})$$





*That's all Folks!*

- Next week

<>

Section 6-6  
Section 6-7  
Section 6-8