ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Circuits and Systems I

LECTURE #12
Frequency Response of FIR

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LIONS/Laboratory for Information and Inference Systems

License Info for SPFirst Slides

Outline - Today

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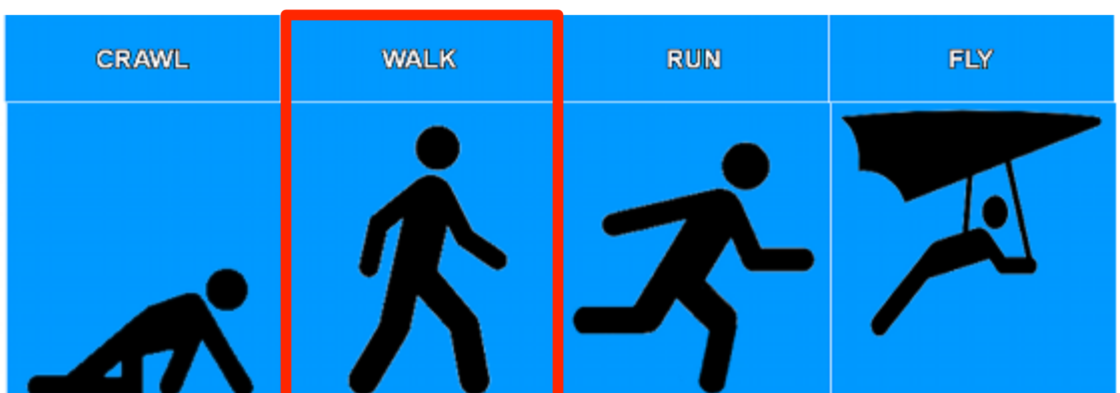
***CSI* Progress Level:**



Lecture Objectives

- SINUSOIDAL INPUT SIGNAL
  - DETERMINE the FIR FILTER OUTPUT
- FREQUENCY RESPONSE of FIR
  - PLOTTING vs. Frequency
  - MAGNITUDE vs. Freq
  - PHASE vs. Freq

MAG

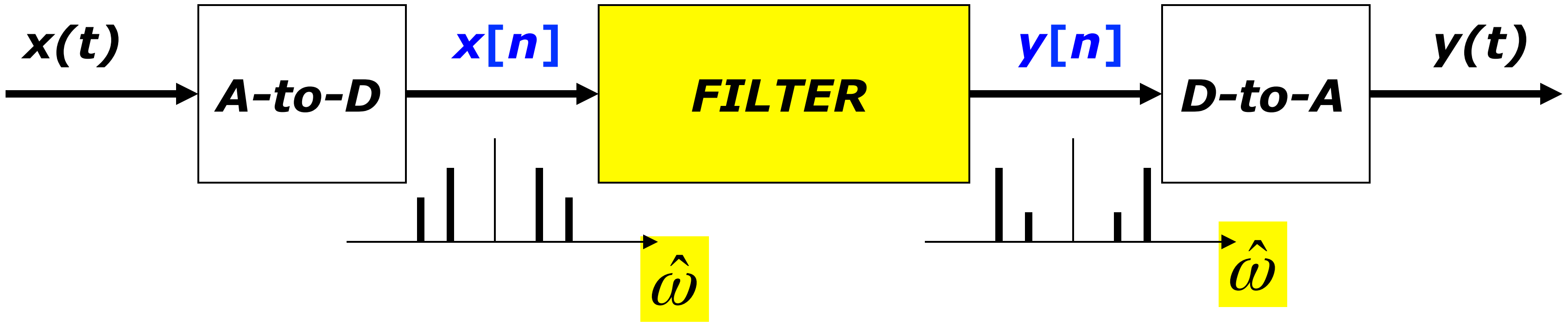
PHASE

$$H(e^{j\hat{\omega}}) = \left|H(e^{j\hat{\omega}})\right| e^{j\angle H(e^{j\hat{\omega}})}$$

DOMAINS: Time & Frequency

- **Time-Domain: "n" = time**
  - x[n]   discrete-time signal
  - x(t) continuous-time signal
- **Frequency Domain (sum of sinusoids)**
  - Spectrum vs. f (Hz)
    - ANALOG vs. DIGITAL
  - Spectrum vs. omega-hat
- Move back and forth QUICKLY

Digital “Filtering”



- CONCENTRATE on the **SPECTRUM**
- SINUSOIDAL INPUT
  - INPUT x[n] = SUM of SINUSOIDS
  - Then, OUTPUT y[n] = SUM of SINUSOIDS

Filtering Example

- 7-point AVERAGER
  - Removes cosine
    - By making its amplitude (A) smaller

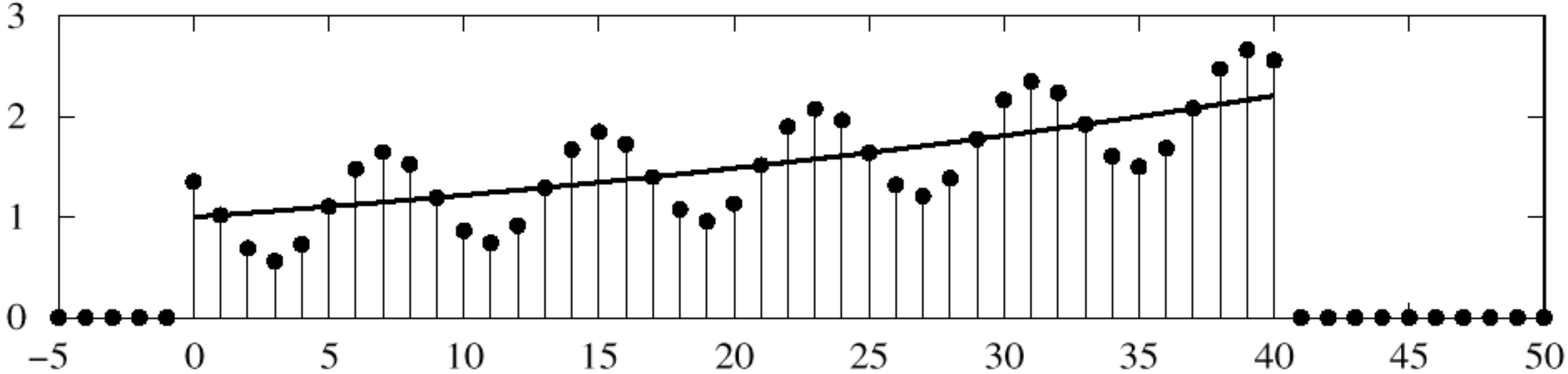
$$y_7[n] = \sum_{k=0}^{6} \left(\tfrac{1}{7}\right) x[n-k]$$

- 3-point AVERAGER
  - Changes A slightly

$$y_3[n] = \sum_{k=0}^{2} \left(\tfrac{1}{3}\right) x[n-k]$$

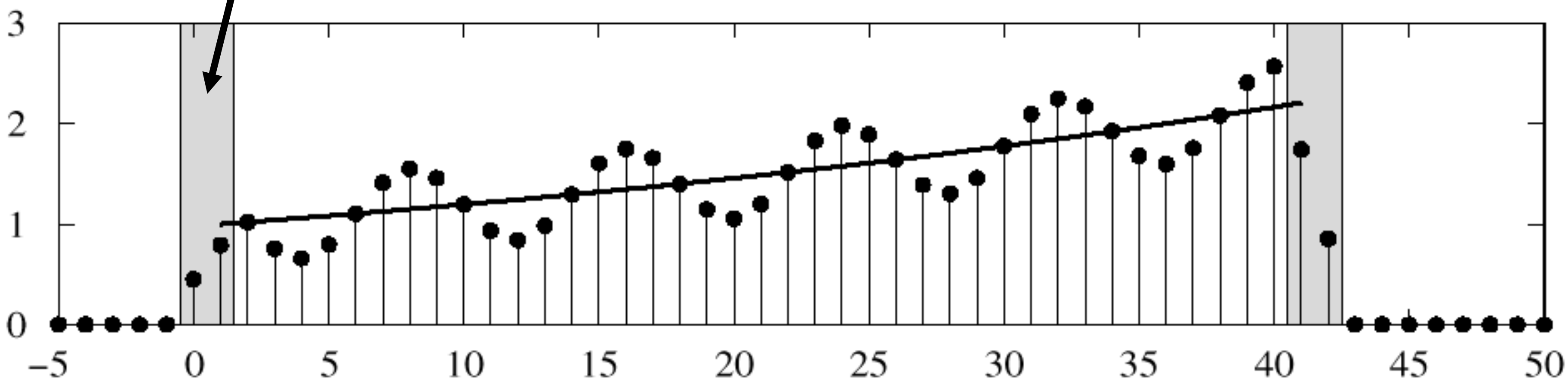
3-pt Averager Example

$$\text{Input}: x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$$



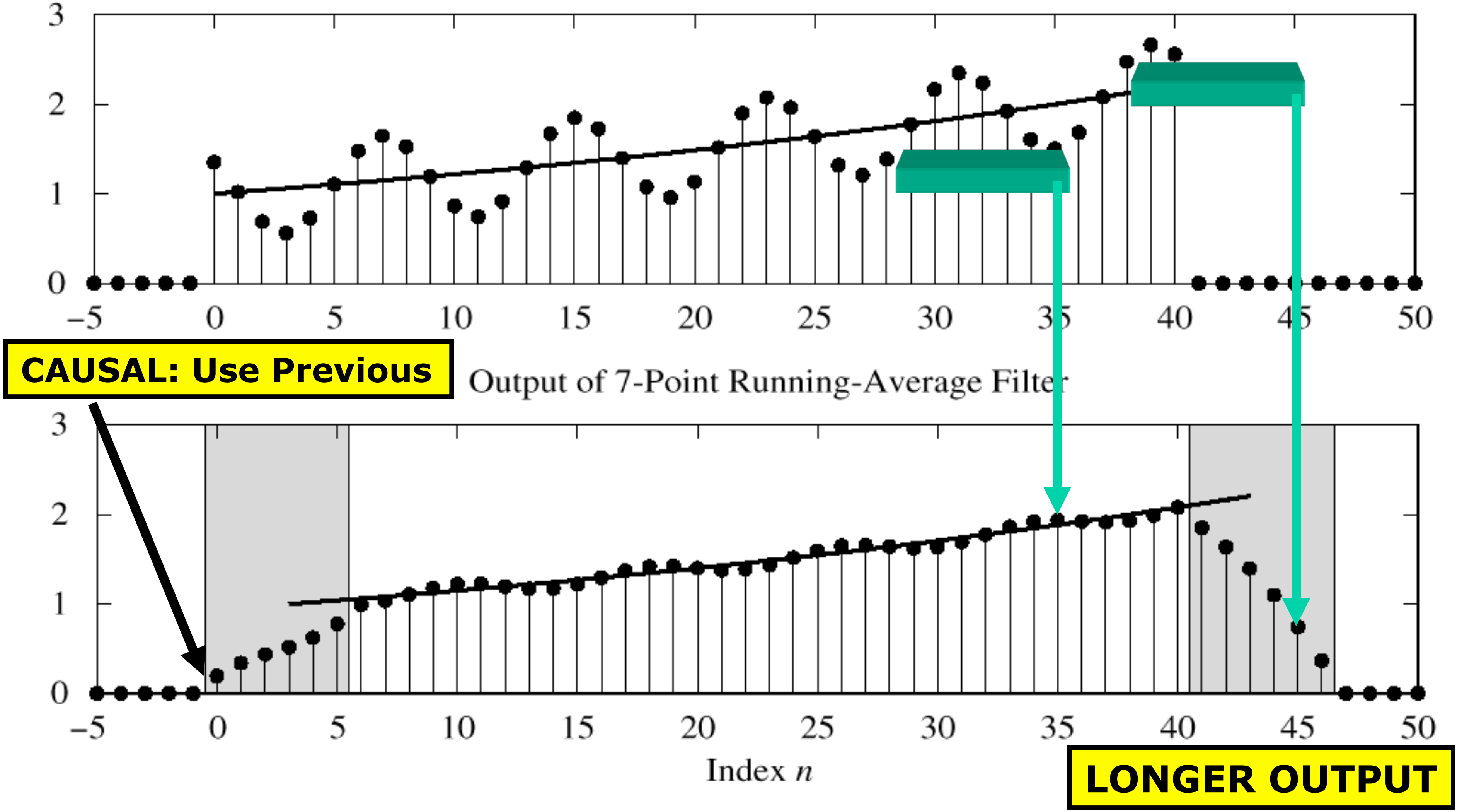
USE PAST VALUES

Output of 3-Point Running-Average Filter



7-pt FIR Example (FIR)

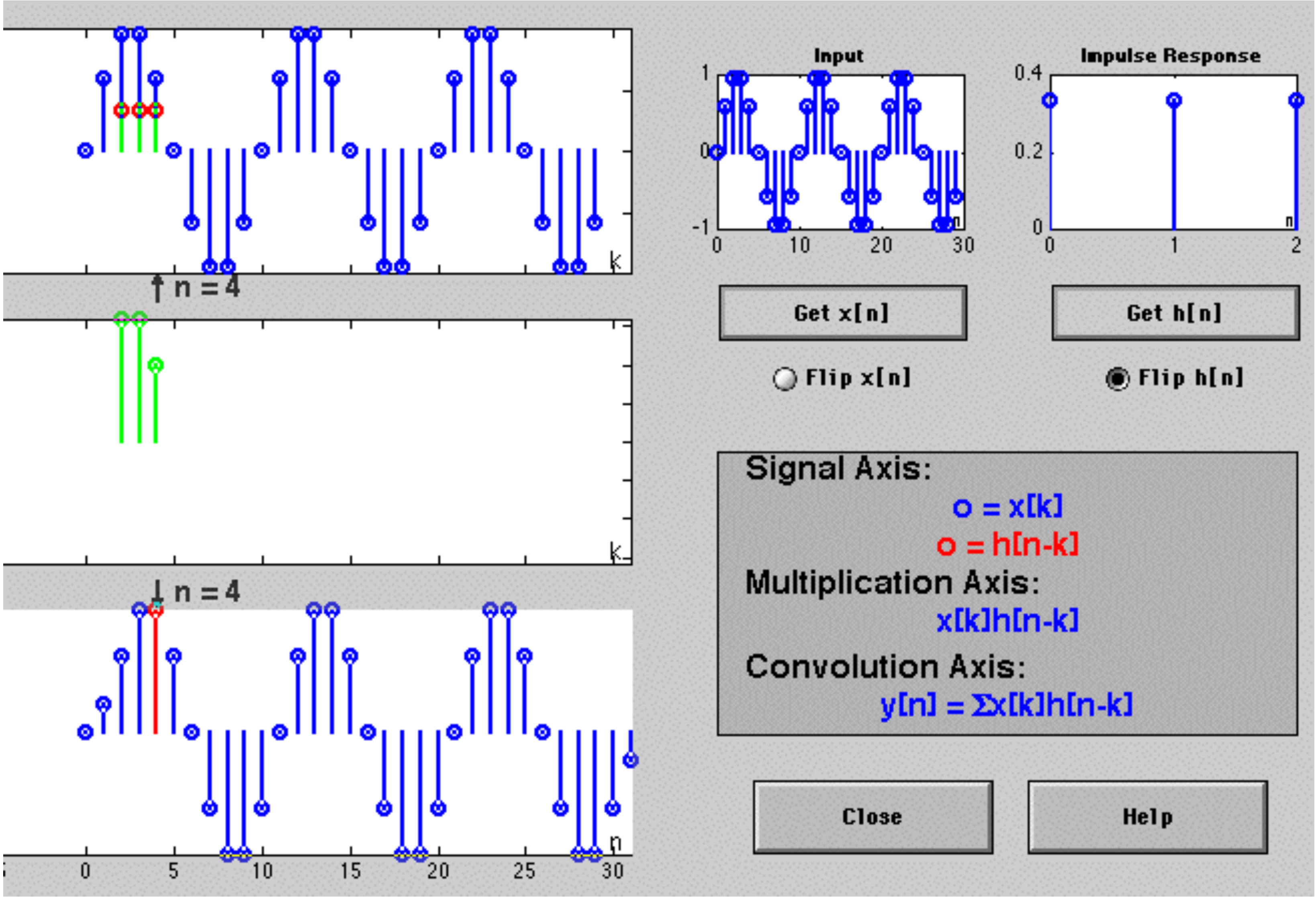
$$\text{Input}: x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$$



Sinusoidal Response

- INPUT: x[n] = SINUSOID
- OUTPUT: y[n] will also be a SINUSOID
  - Different Amplitude and Phase
  - **SAME** Frequency
- AMPLITUDE & PHASE CHANGE
  - Called the **FREQUENCY RESPONSE**

DCONVDEMO: MATLAB GUI



Complex Exponential

$$x[n] = Ae^{j\varphi}e^{j\hat{\omega}n} \qquad -\infty < n < \infty$$

x[n] is the input signal—a complex exponential

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k]x[n-k]$$

FIR DIFFERENCE EQUATION

Complex Exponential Output

- Use the FIR “Difference Equation”

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} b_k A e^{j\varphi} e^{j\hat{\omega}(n-k)}$$

$$= \left(\sum_{k=0}^{M} b_k e^{j\hat{\omega}(-k)} \right) A e^{j\varphi} e^{j\hat{\omega} n}$$

$$= H(\hat{\omega}) A e^{j\varphi} e^{j\hat{\omega} n}$$

Frequency Response

- At each frequency, we can DEFINE

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

FREQUENCY RESPONSE

- Complex-valued formula
  - Has MAGNITUDE vs. frequency
  - And PHASE vs. frequency
- Notation: $H(e^{j\hat{\omega}})$ in place of $H(\hat{\omega})$

EXAMPLE 6.1

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

$$\{b_k\} = \{1,\ 2,\ 1\}$$

$$H(e^{j\hat{\omega}}) = 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$$

$$= e^{-j\hat{\omega}}(e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}})$$

$$= e^{-j\hat{\omega}}(2 + 2\cos\hat{\omega})$$

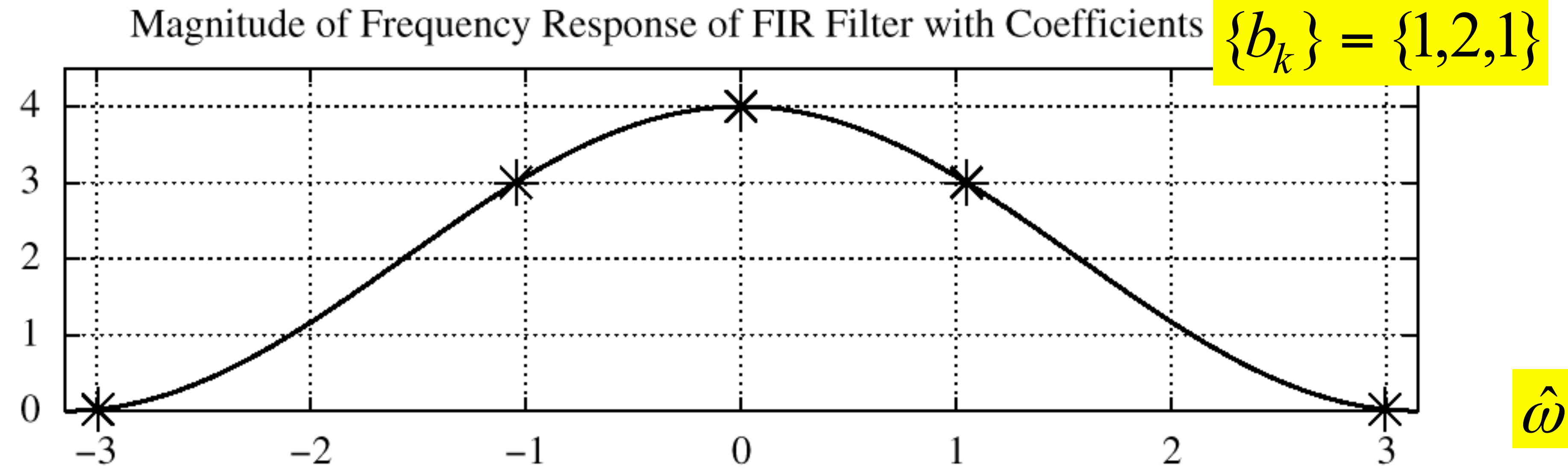
EXPLOIT SYMMETRY

Since $(2 + 2\cos\hat{\omega}) \geq 0$

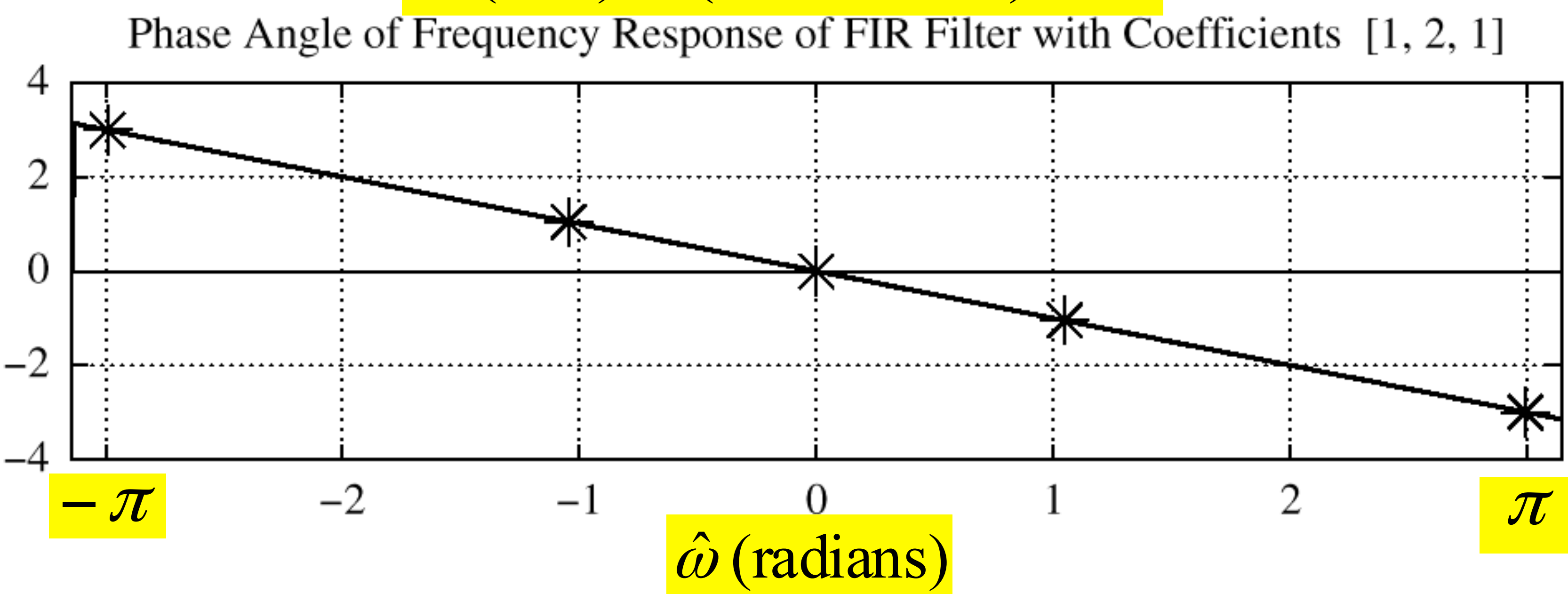
Magnitude is $\left|H(e^{j\hat{\omega}})\right| = (2 + 2\cos\hat{\omega})$

and Phase is $\angle H(e^{j\hat{\omega}}) = -\hat{\omega}$

PLOT of FREQ RESPONSE

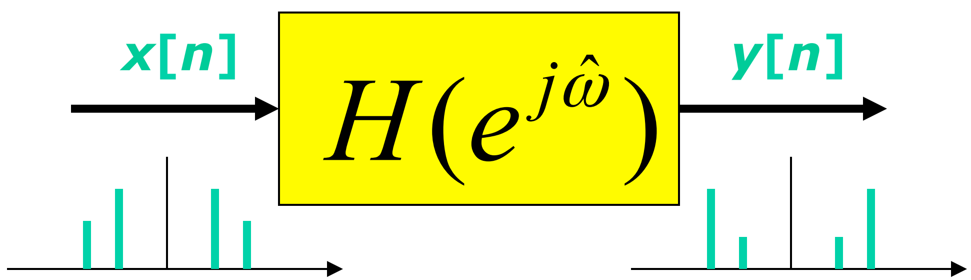


$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$



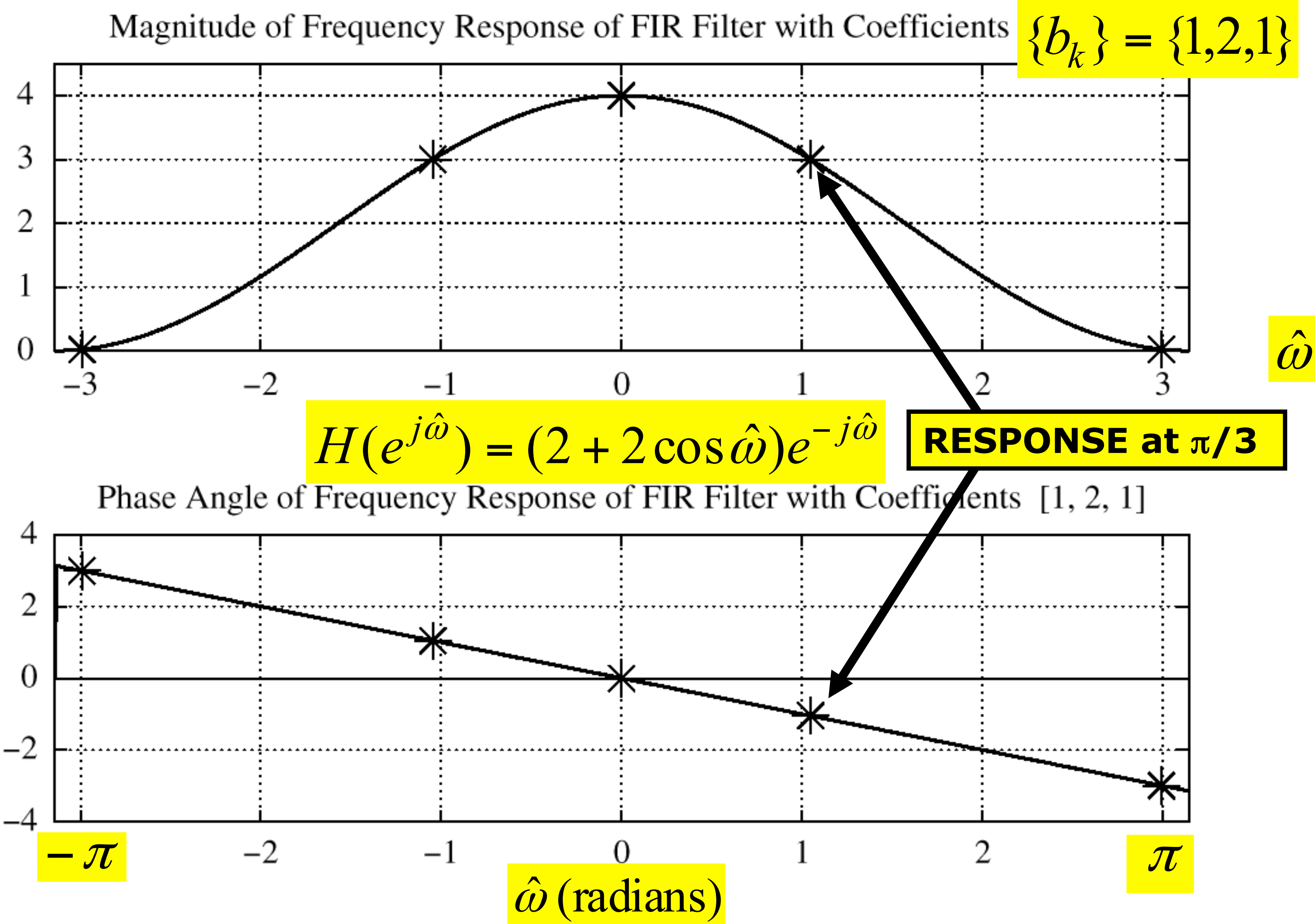
EXAMPLE 6.2

Find $y[n]$ when $H(e^{j\hat{\omega}})$ is known and $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$



$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

PLOT of FREQ RESPONSE



EXAMPLE 6.2 (answer)

$$\text{Find } y[n] \text{ when } x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$$

$$\text{One Step - evaluate } H(e^{j\hat{\omega}}) \text{ at } \hat{\omega} = \pi/3$$

$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

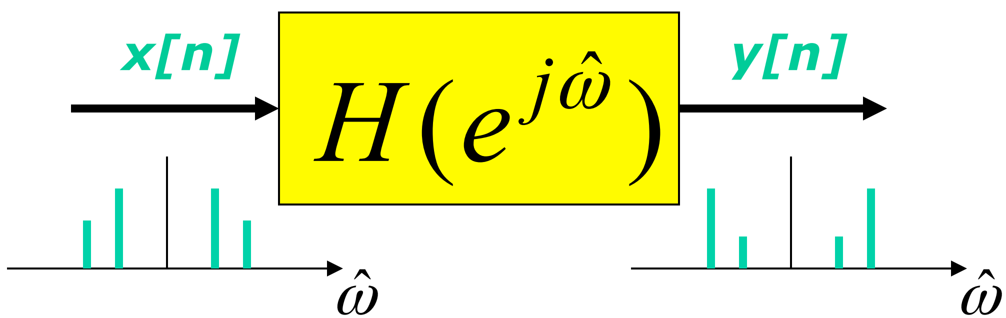
$$H(e^{j\hat{\omega}}) = 3e^{-j\pi/3} \quad @\,\hat{\omega} = \pi/3$$

$$y[n] = \left(3e^{-j\pi/3}\right) \times 2e^{j\pi/4}e^{j(\pi/3)n} = 6\,e^{-j\pi/12}e^{j(\pi/3)n}$$

Example: COSINE Input

Find $y[n]$ when $H(e^{j\hat{\omega}})$ is known

and $x[n] = 2\cos(\frac{\pi}{3}n + \frac{\pi}{4})$



$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

EX: COSINE Input

$$\text{Find } y[n] \text{ when } x[n] = 2\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{4})$$

$$2\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{4}) = e^{j(\pi n/3+\pi/4)} + e^{-j(\pi n/3+\pi/4)}$$

$$\Rightarrow x[n] = x_1[n] + x_2[n]$$

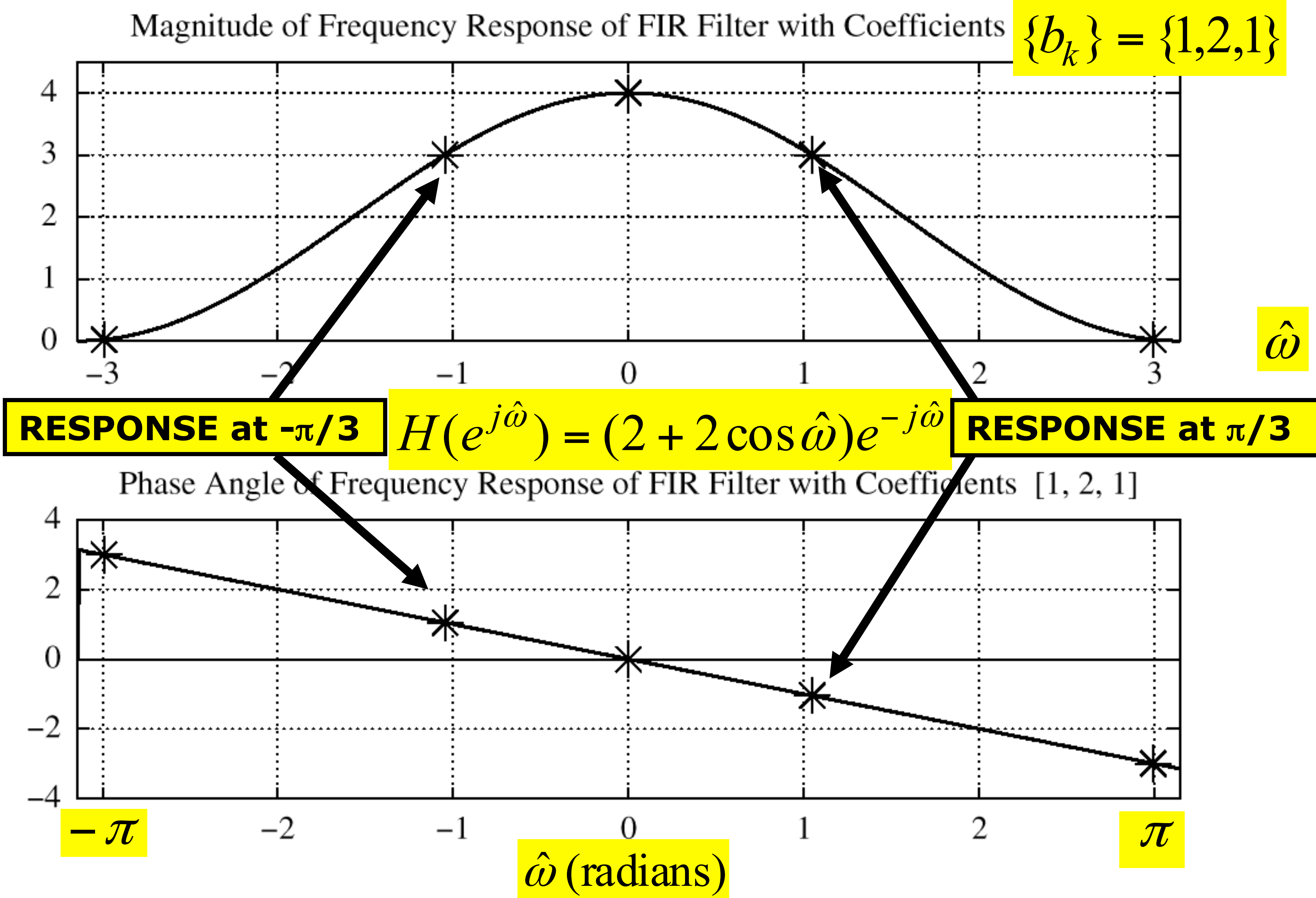
Use Linearity

$$y_1[n] = H(e^{j\pi/3})e^{j(\pi n/3+\pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3})e^{-j(\pi n/3+\pi/4)}$$

$$\Rightarrow y[n] = y_1[n] + y_2[n]$$

PLOT of FREQ RESPONSE



EX: COSINE Input (ans-2)

$$\text{Find } y[n] \text{ when } x[n] = 2\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{4})$$

$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

$$y_1[n] = H(e^{j\pi/3})e^{j(\pi n/3+\pi/4)} = 3e^{-j(\pi/3)}e^{j(\pi n/3+\pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3})e^{-j(\pi n/3+\pi/4)} = 3e^{j(\pi/3)}e^{-j(\pi n/3+\pi/4)}$$

$$y[n] = 3e^{j(\pi n/3-\pi/12)} + 3e^{-j(\pi n/3-\pi/12)}$$

$$\Rightarrow y[n] = 6\cos(\tfrac{\pi}{3}n - \tfrac{\pi}{12})$$

MATLAB: Frequency Response

- `HH = freqz(bb,1,ww)`
  - VECTOR **bb** contains Filter Coefficients
  - SP-First: `HH = freekz(bb,1,ww)`
- FILTER COEFFICIENTS $\{b_k\}$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

LTI Systems

- LTI: Linear & Time-Invariant
- COMPLETELY CHARACTERIZED by:
  - **FREQUENCY RESPONSE**, or
  - IMPULSE RESPONSE h[n]
- **Sinusoid IN -----> Sinusoid OUT**
  - **At the SAME Frequency**

Time & Frequency Relation

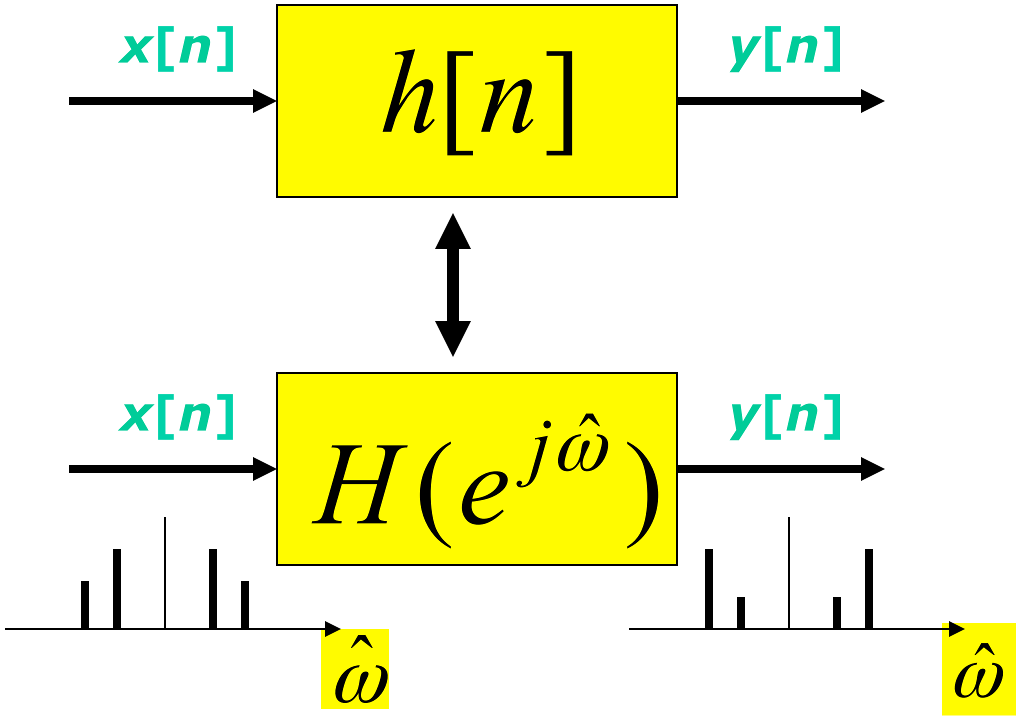
- Get Frequency Response from h[n]
  - Here is the FIR case:

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} = \sum_{k=0}^{M} h[k] e^{-j\hat{\omega}k}$$

IMPULSE RESPONSE

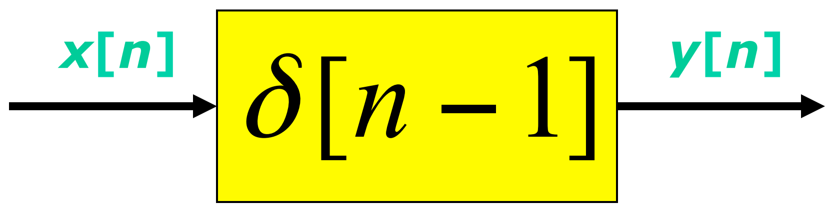
Block Diagrams

- Equivalent Representations

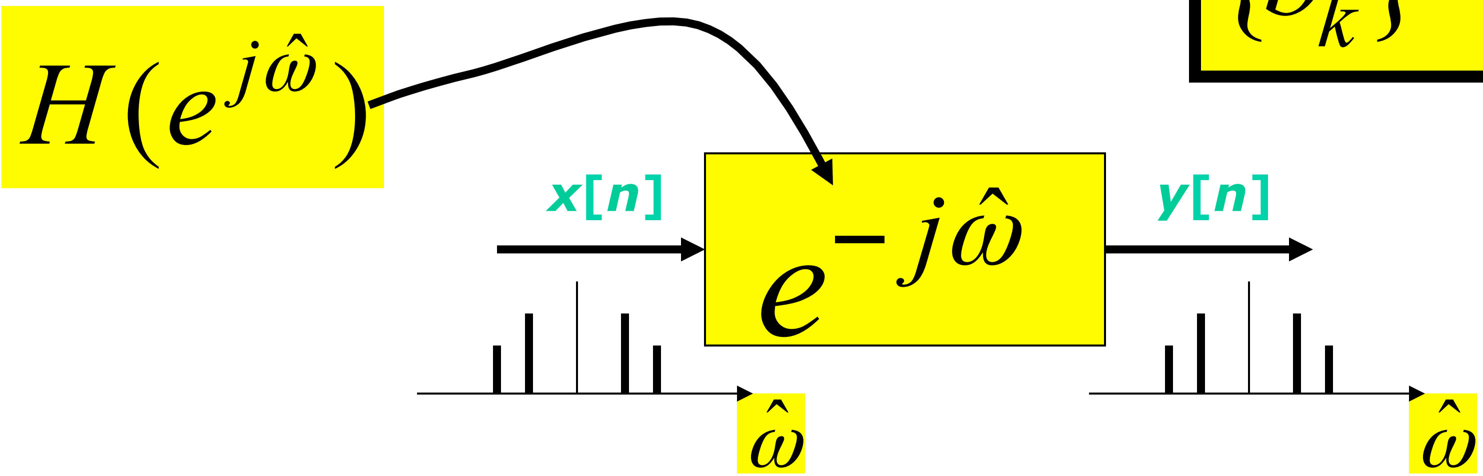


Unit-Delay System

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-1]$

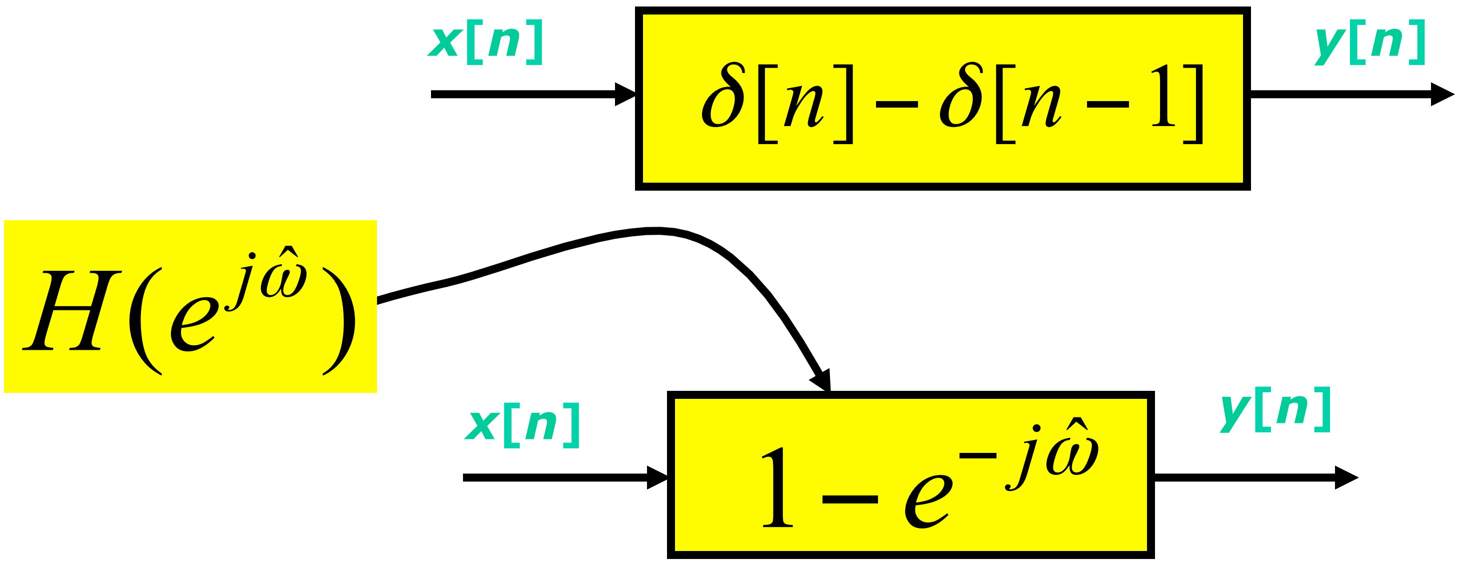


$$\{b_k\} = \{\,0,\ 1\,\}$$

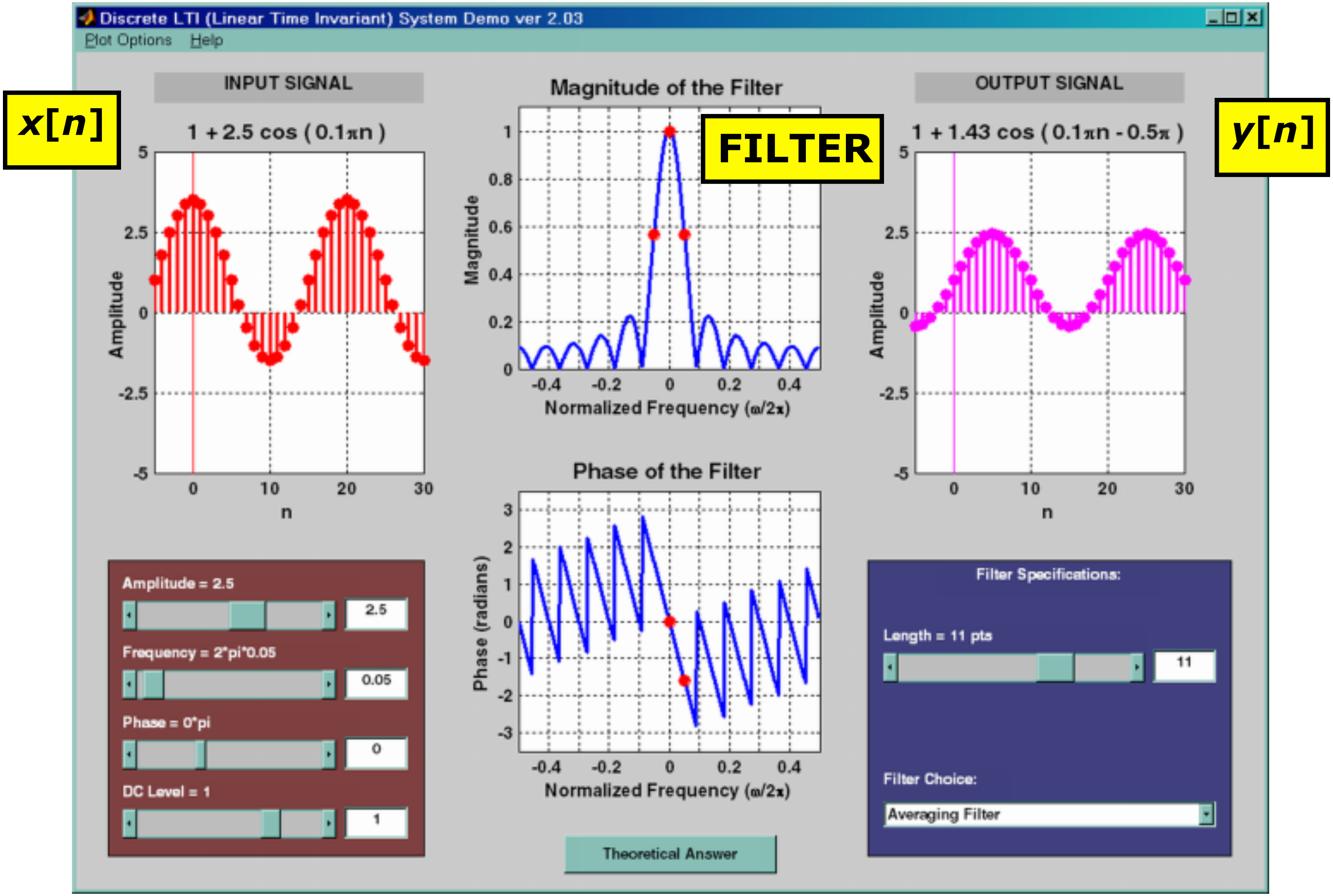


First Difference System

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for the Difference Equation: $y[n] = x[n] - x[n-1]$

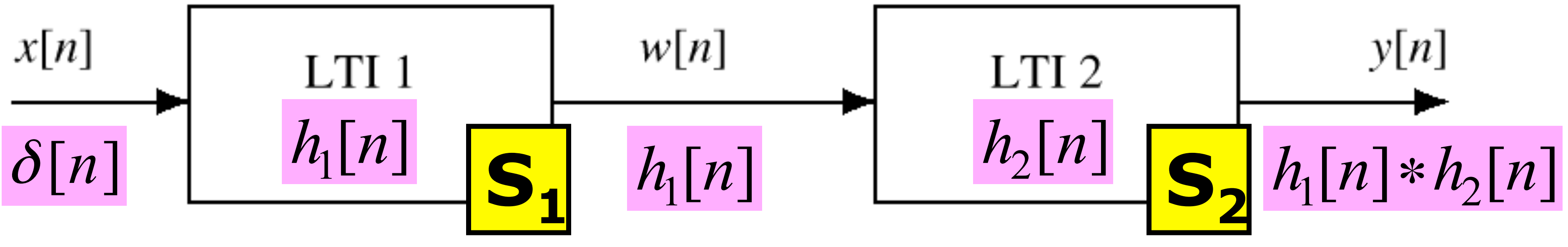


DLTI Demo with Sinusoids



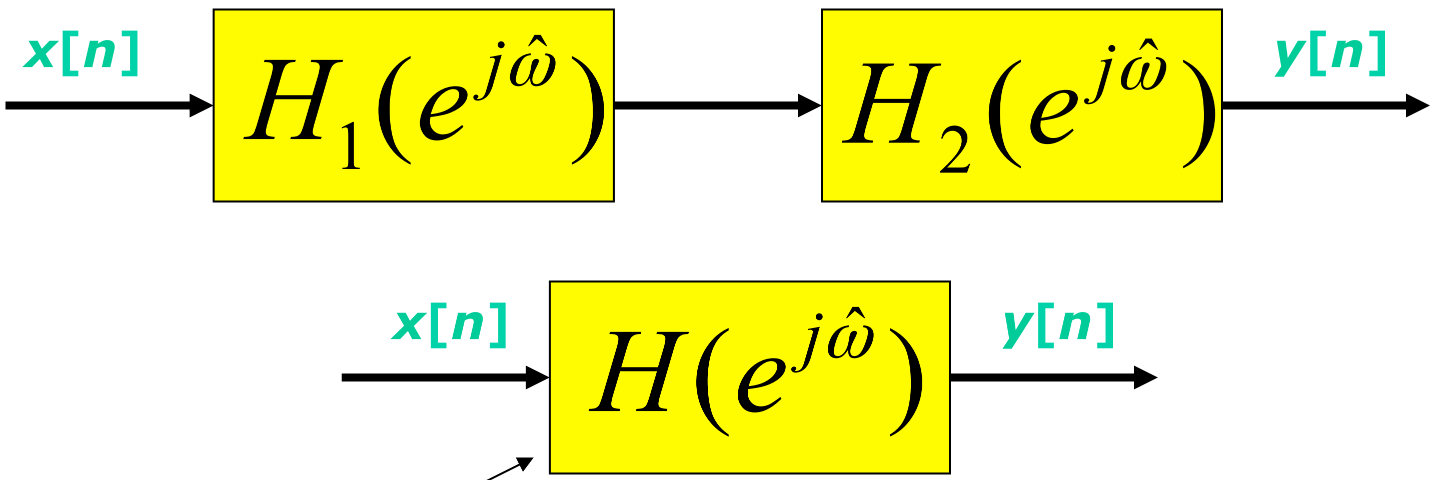
Cascade Systems

- Does the order of S_1 & S_2 matter?
  - NO, LTI SYSTEMS can be rearranged !!!
  - WHAT ARE THE FILTER COEFFS? $\{b_k\}$
  - WHAT is the overall FREQUENCY RESPONSE ?



Cascade Equivalent

- MULTIPLY the Frequency Responses



EQUIVALENT SYSTEM

$$H(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}})H_2(e^{j\hat{\omega}})$$

- Next week <> Section 6-6
  Section 6-7
  Section 6-8