ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

# *Circuits and Systems I*

LECTURE #13

## Frequency Response of FIR Filters and Digital Filtering of Analog Signals

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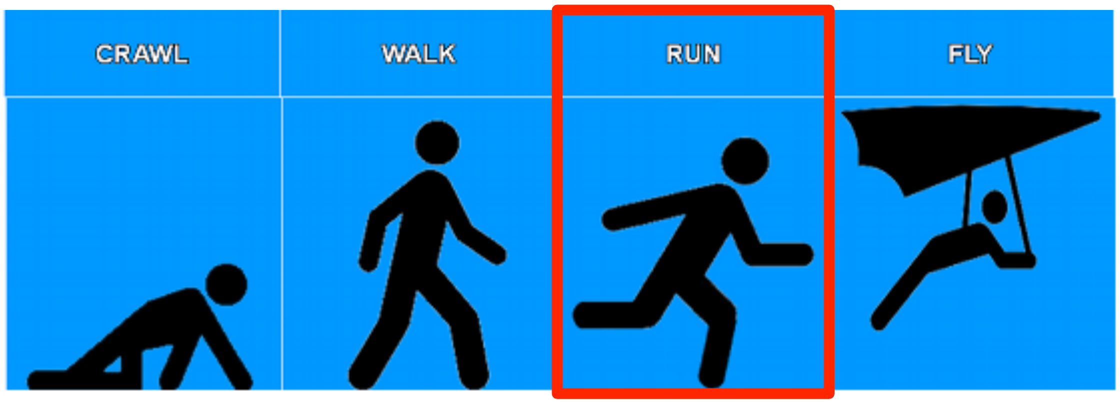
# License Info for SPFirst Slides

# Outline - Today

- Today <> Section 6-6
  Section 6-7
  Section 6-8

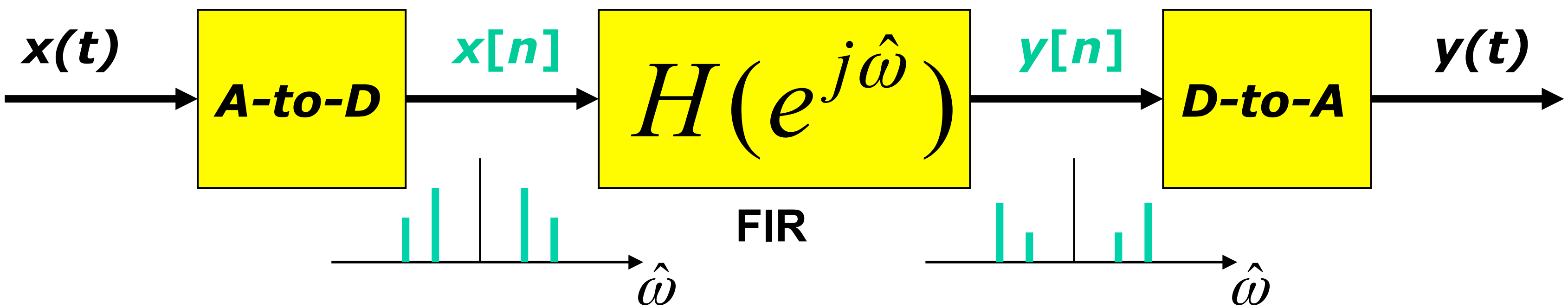
- Next week <> Final Exam Review

***CSI* Progress Level:**



# LECTURE OBJECTIVES

- Two Domains: Time & Frequency
- Track the spectrum of x[n] thru an FIR Filter: **Sinusoid-IN gives Sinusoid-OUT**
- **UNIFICATION**: How does frequency response affect x(t) to produce y(t) ?



# TIME & FREQUENCY

**FIR DIFFERENCE EQUATION is the TIME-DOMAIN**

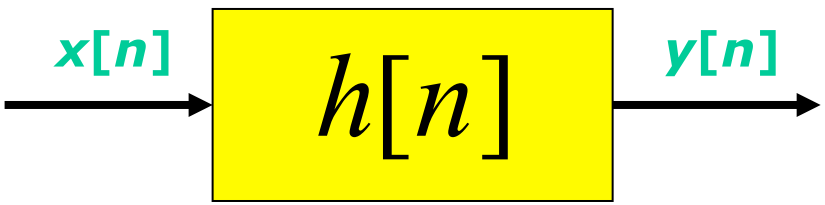
$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k]x[n-k]$$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} h[k] e^{-j\hat{\omega}k}$$

$$H(e^{j\hat{\omega}}) = h[0] + h[1]e^{-j\hat{\omega}} + h[2]e^{-j2\hat{\omega}} + h[3]e^{-j3\hat{\omega}} + \cdots$$

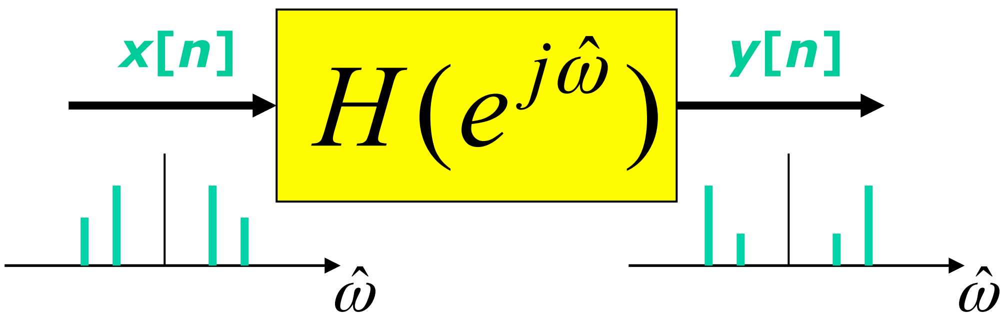
# Ex: DELAY by 2 SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-2]$



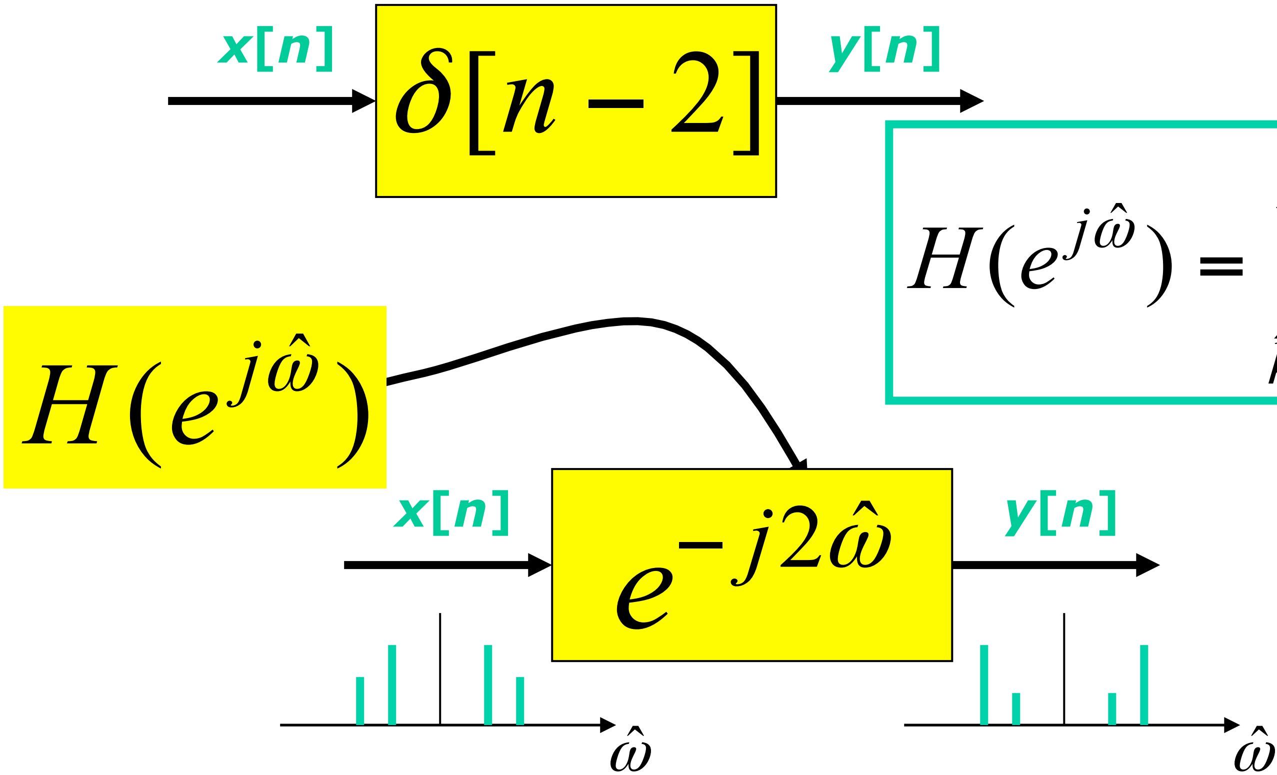

$$b_k = \{\, 0, 0, 1 \,\}$$

$$h[n] = \delta[n-2]$$



# DELAY by 2 SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-2]$




$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} \delta[k-2]e^{-j\hat{\omega}k}$$

***k* = 2 ONLY**

# GENERAL DELAY PROPERTY

$$\text{Find } h[n] \text{ and } H(e^{j\hat{\omega}}) \text{ for } y[n] = x[n - n_d]$$

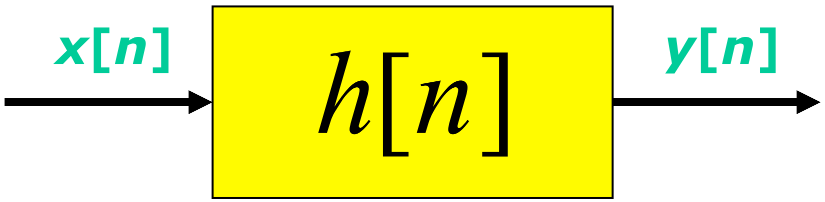
$$h[n] = \delta[n - n_d]$$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} \delta[k - n_d] e^{-j\hat{\omega}k} = e^{-j\hat{\omega}n_d}$$

**ONLY ONE non-ZERO TERM for *k* at *k* = $n_d$**

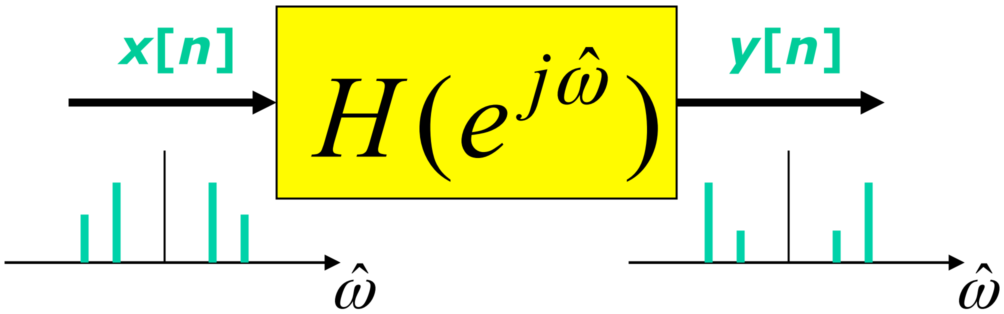
# FREQ DOMAIN --> TIME ??

- START with $H(e^{j\hat{\omega}})$ and find $h[n]$ or $b_k$




$$h[n] = ?$$

$$H(e^{j\hat{\omega}}) = 7e^{-j2\hat{\omega}}\cos(\hat{\omega})$$



# FREQ DOMAIN --> TIME

$$H(e^{j\hat{\omega}}) = 7e^{-j2\hat{\omega}}\cos(\hat{\omega})$$

**EULER's Formula**

$$= 7e^{-j2\hat{\omega}}(0.5e^{j\hat{\omega}} + 0.5e^{-j\hat{\omega}})$$

$$= (3.5e^{-j\hat{\omega}} + 3.5e^{-j3\hat{\omega}})$$

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$$h[n] = 3.5\delta[n-1] + 3.5\delta[n-3]$$

$$b_k = \{\,0,\,3.5,\,0,3.5\,\}$$

# PREVIOUS LECTURE REVIEW

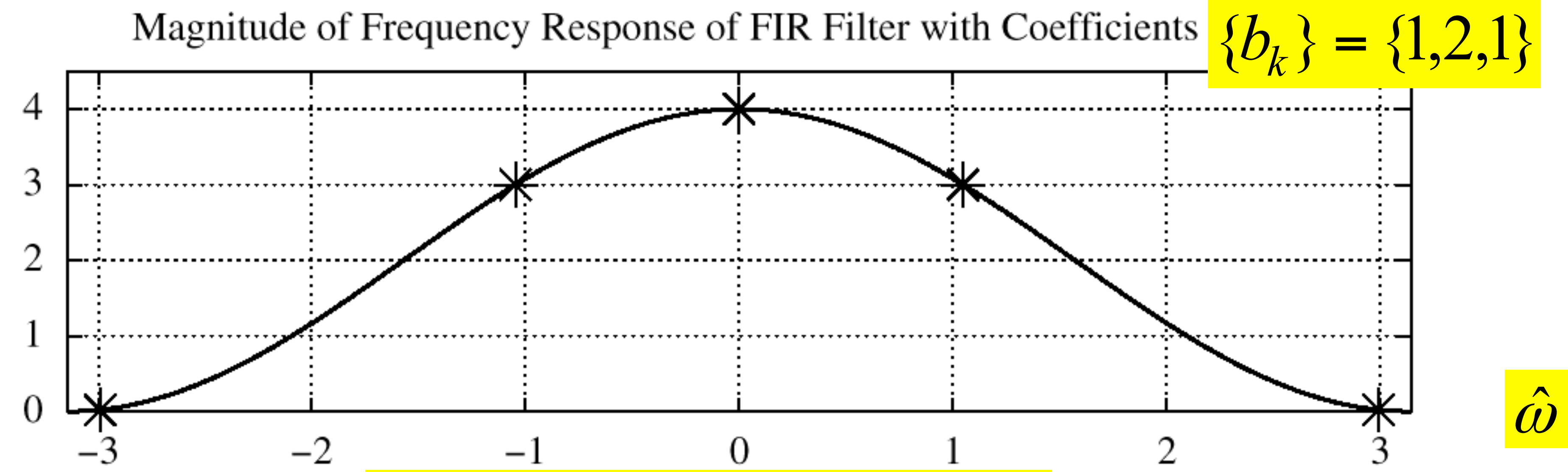
- SINUSOIDAL INPUT SIGNAL
  - OUTPUT has SAME FREQUENCY
  - DIFFERENT Amplitude and Phase
- FREQUENCY RESPONSE of FIR
  - MAGNITUDE vs. Frequency
  - PHASE vs. Freq
  - PLOTTING

MAG

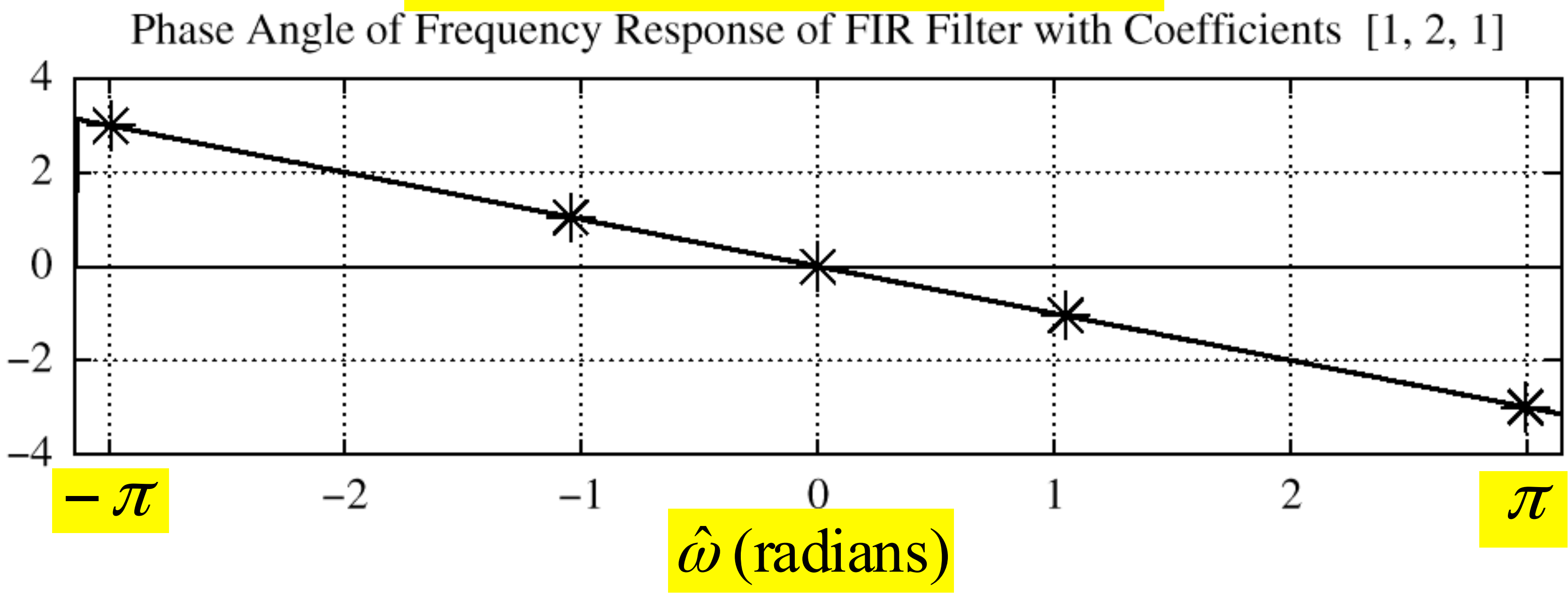
PHASE

$$H(e^{j\hat{\omega}}) = \left|H(e^{j\hat{\omega}})\right| e^{j\angle H(e^{j\hat{\omega}})}$$

# PLOT of FREQ RESPONSE




$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$



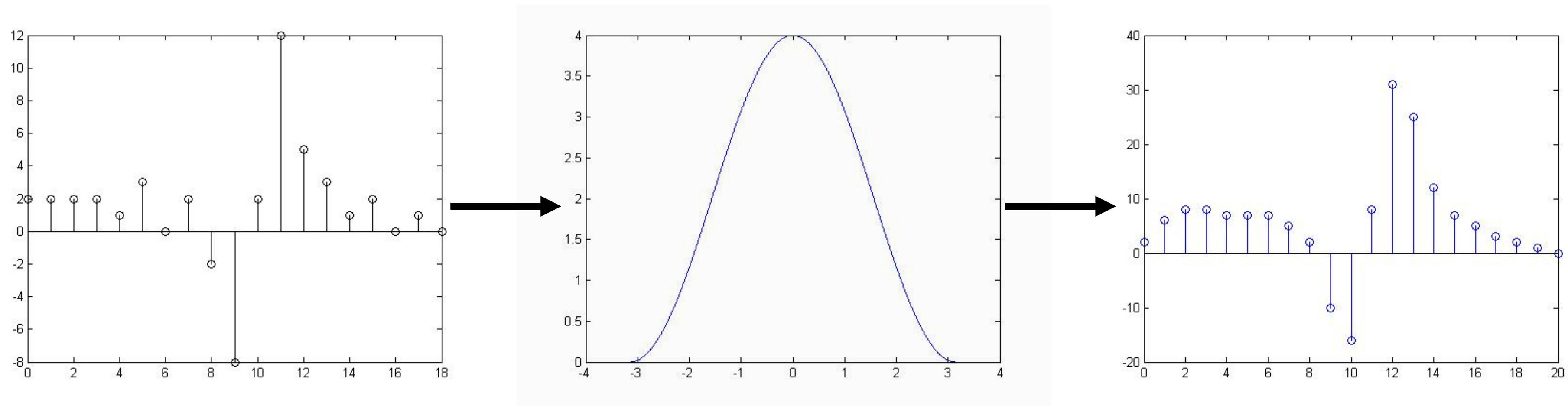
# FILTER TYPES

- LOW-PASS FILTER (**LPF**)
  - BLURRING
  - ATTENUATES HIGH FREQUENCIES
- HIGH-PASS FILTER (**HPF**)
  - SHARPENING for IMAGES
  - BOOSTS THE HIGHS
  - REMOVES DC
- BAND-PASS FILTER (**BPF**)

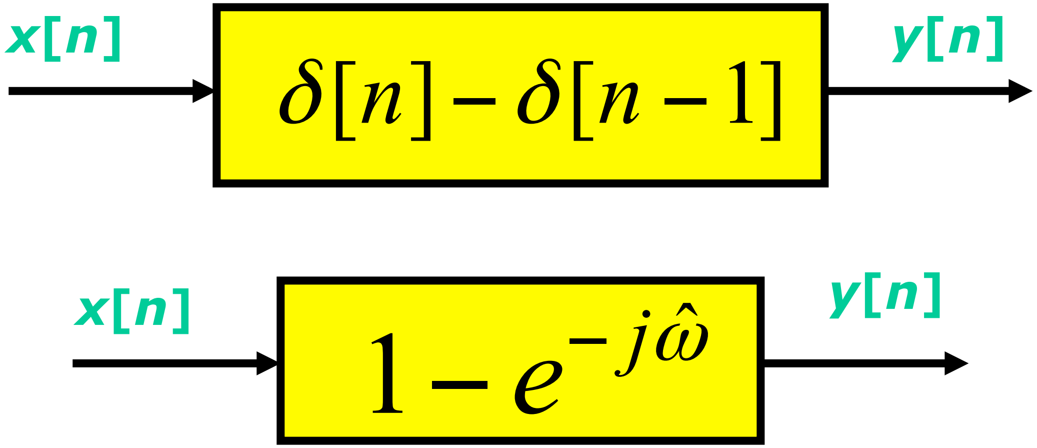
# LOW-PASS FILTER EXAMPLE

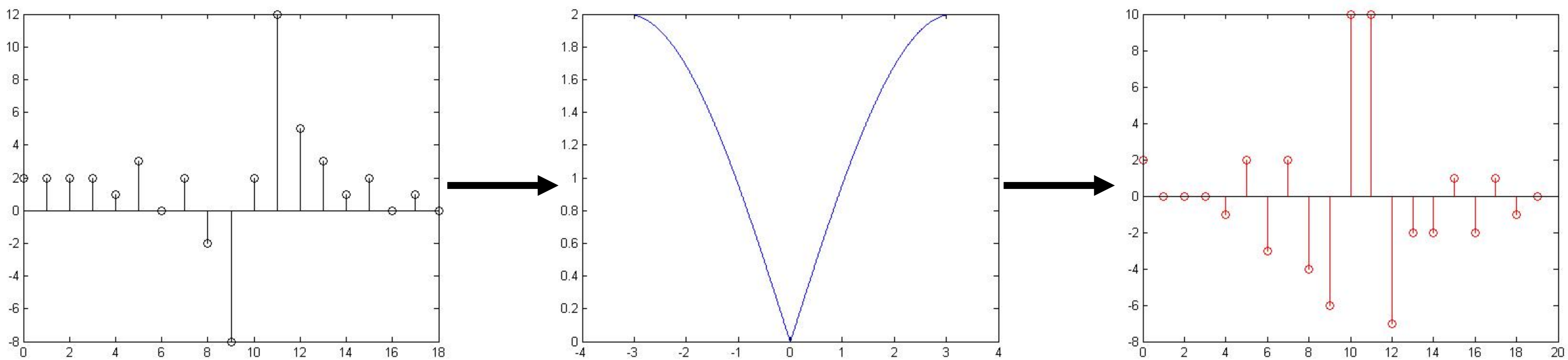
$x[n]$ → $\delta[n] + 2\delta[n-1] + \delta[n-2]$ → $y[n]$

$x[n]$ → $1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$ → $y[n]$



# HIGH-PASS FILTER EXAMPLE



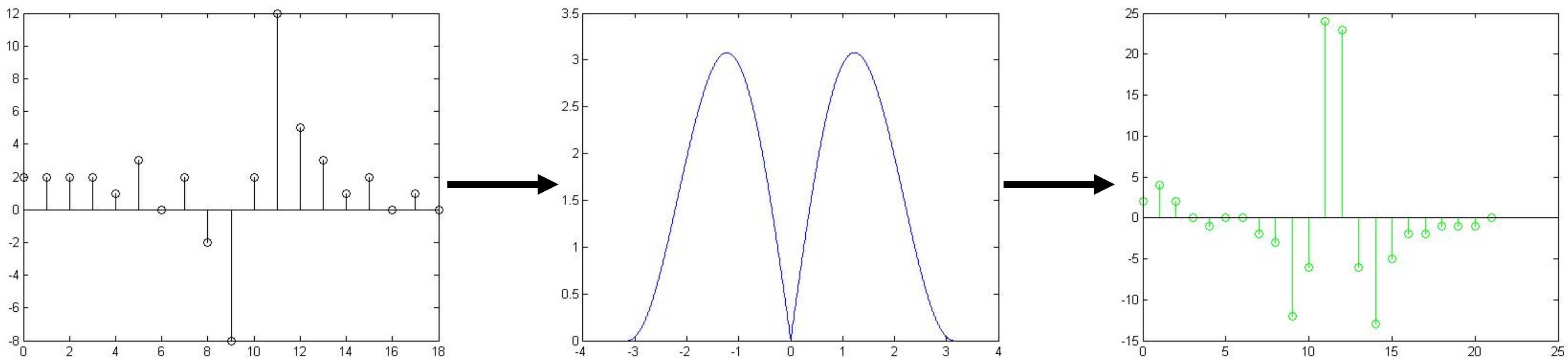



# BAND-PASS FILTER EXAMPLE

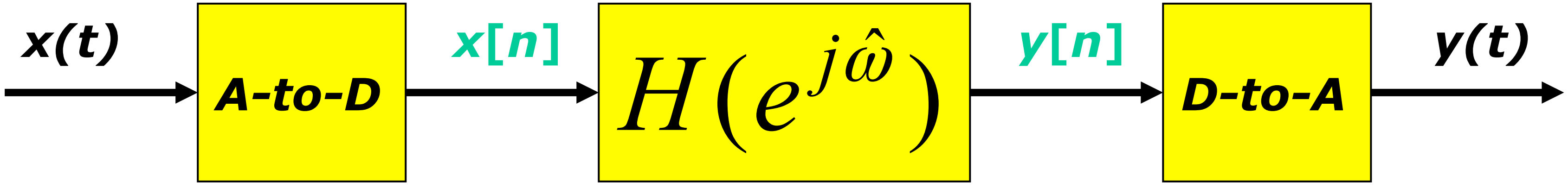
$x[n] \rightarrow$ $\delta[n] + 2\delta[n-1] + \delta[n-2]$ $\rightarrow$ $\delta[n] - \delta[n-1]$ $\rightarrow y[n]$

$x[n] \rightarrow$ $1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$ $\rightarrow$ $1 - e^{-j\hat{\omega}}$ $\rightarrow y[n]$

$x[n] \rightarrow$ $1 + e^{-j\hat{\omega}} - e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$ $\rightarrow y[n]$

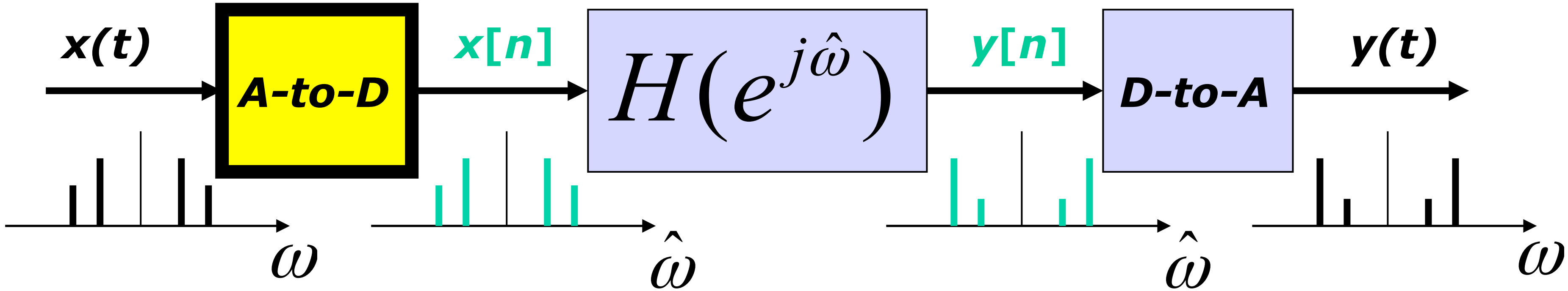


# DIGITAL FILTERING OF ANALOG SIGNALS




- Use discrete-time filters to filter continuous-time signals that have been sampled
- What is the effect of the filter on the continuous-time input x(t)?
- What is the equivalent analog frequency response?

# FREQUENCY SCALING

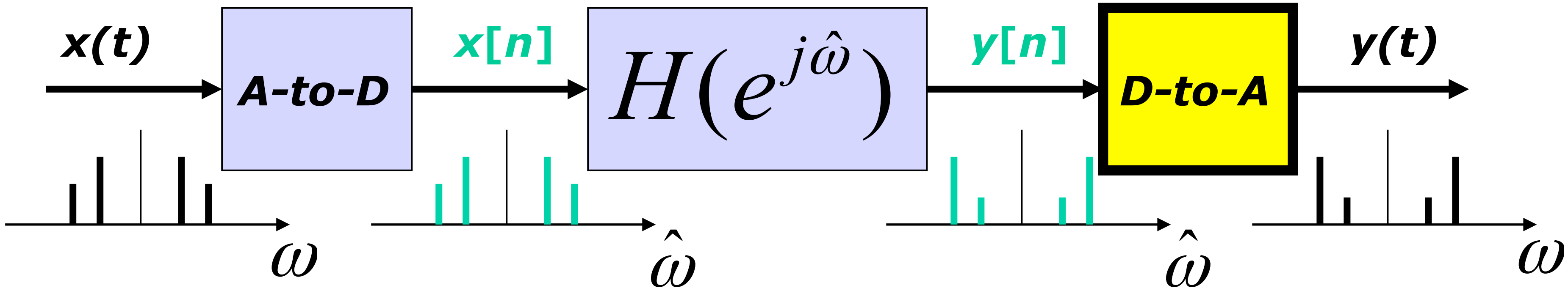



$$x[n] = x(nT_s)$$

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s} = \frac{2\pi f}{f_s}$$

- If $|\omega| < \pi/T_s$
  - NO ALIASING
  - $|\hat{\omega}| < \pi$

# D-A FREQUENCY SCALING




- TIME SAMPLING:

$$t = nT_s \Rightarrow n \leftarrow t f_s$$

$$\omega = \hat{\omega} f_s$$

- RECONSTRUCT up to $0.5f_s$
  - FREQUENCY SCALING

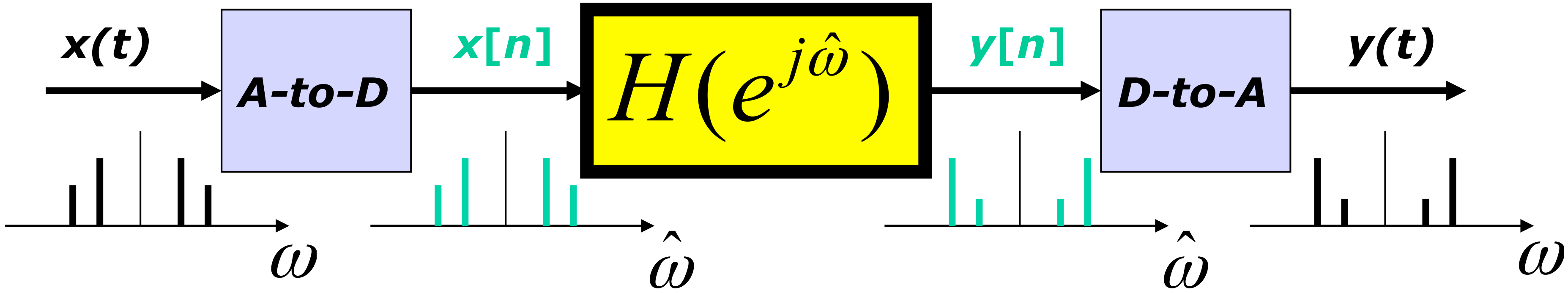
- If input is $x(t) = Ae^{j\phi}e^{j\omega t}$,

output is $y(t) = H(e^{j(\omega T_s)})Ae^{j\phi}e^{j\omega t}$

ANALOG FREQUENCY RESPONSE

for frequencies ω such that $-\pi / T_s < \omega < \pi / T_s$

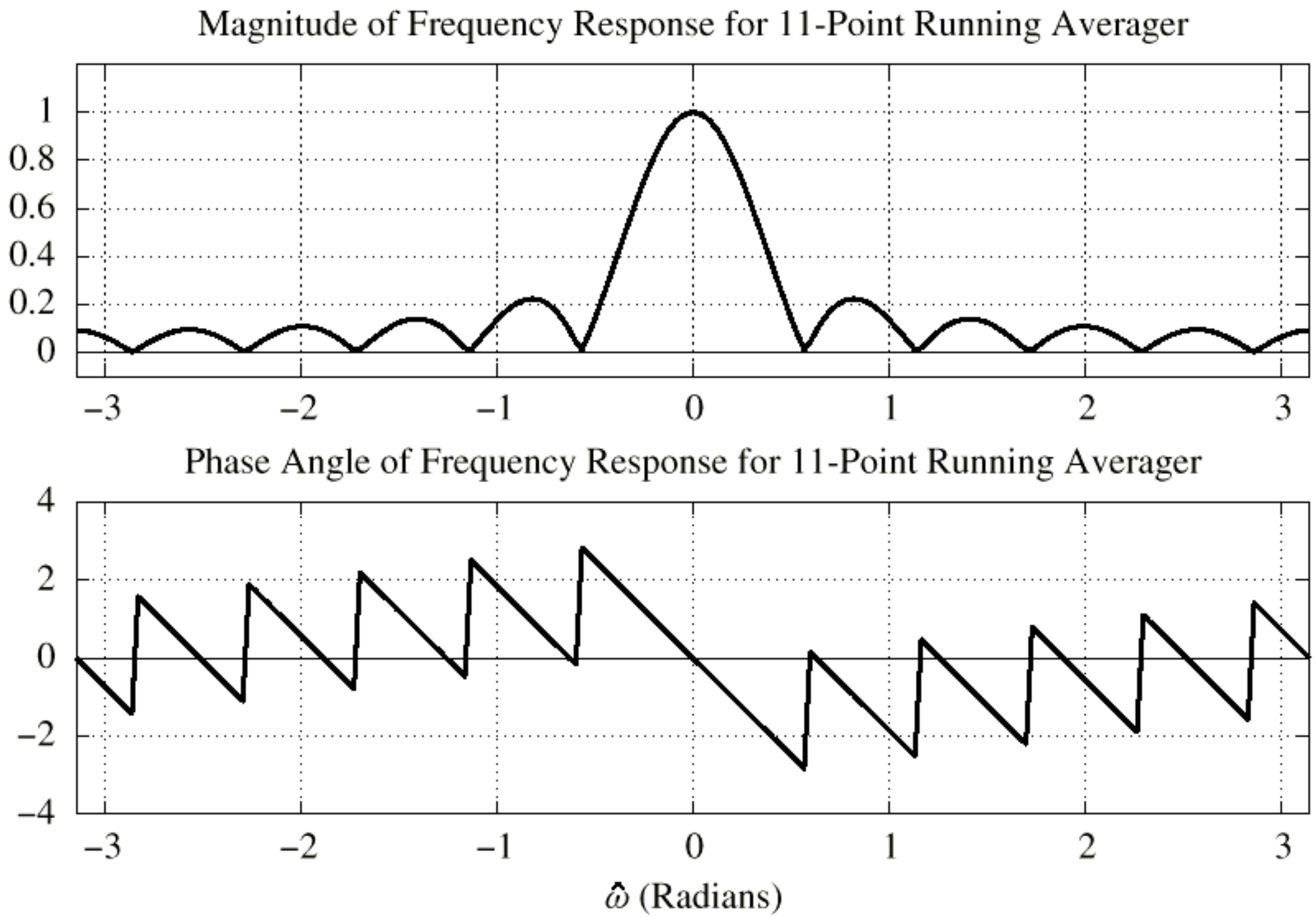
# 11-pt AVERAGER Example



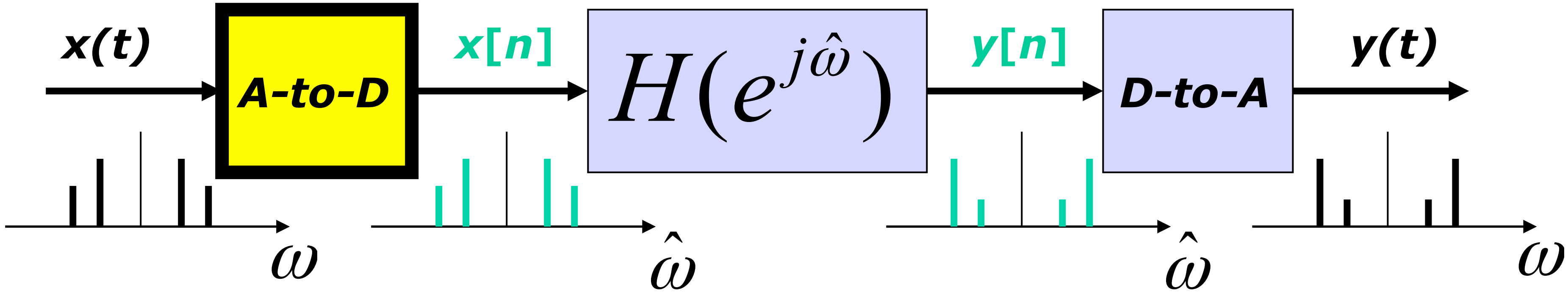

$$y[n] = \sum_{k=0}^{10} \frac{1}{11} x[n-k]$$

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2}\hat{\omega})}{11\sin(\frac{1}{2}\hat{\omega})} e^{-j5\hat{\omega}}$$

# 11-pt AVERAGER



# 11-pt AVERAGER Example



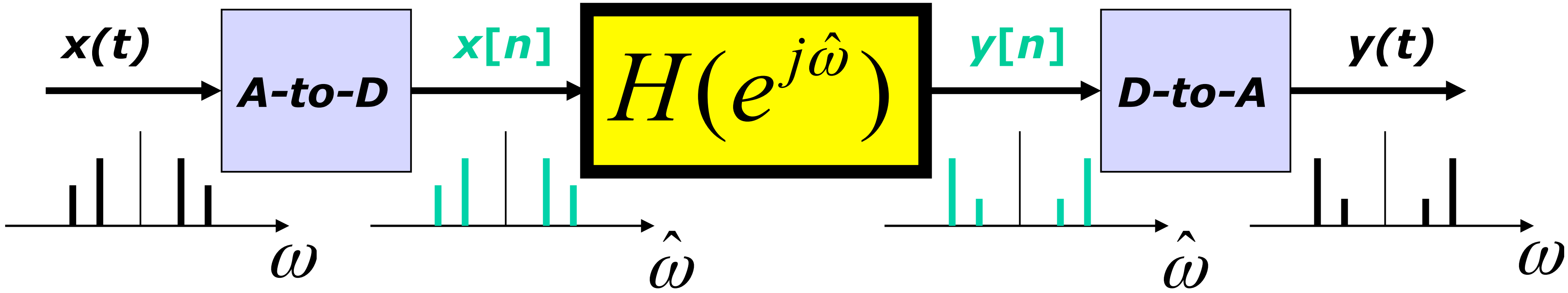

Input: $x(t) = \cos(2\pi(25)t) + \cos(2\pi(250)t - \frac{1}{2}\pi)$

Input frequencies: 25 Hz and 250 Hz

Sampling frequency: $f_s$ = 1000 Hz

Note: $f_s > 2 f_{max}$ so no aliasing and x(t) can be reconstructed from x[n]

# TRACK the FREQUENCIES




| | | | | |
|---|---|---|---|---|
| 250 Hz | $0.5\pi$ | $H(e^{j0.5\pi})$ | $0.5\pi$ | 250 Hz |
| 25 Hz | $0.05\pi$ | $H(e^{j0.05\pi})$ | $0.05\pi$ | 25 Hz |

**$f_s$ = 1000 Hz**

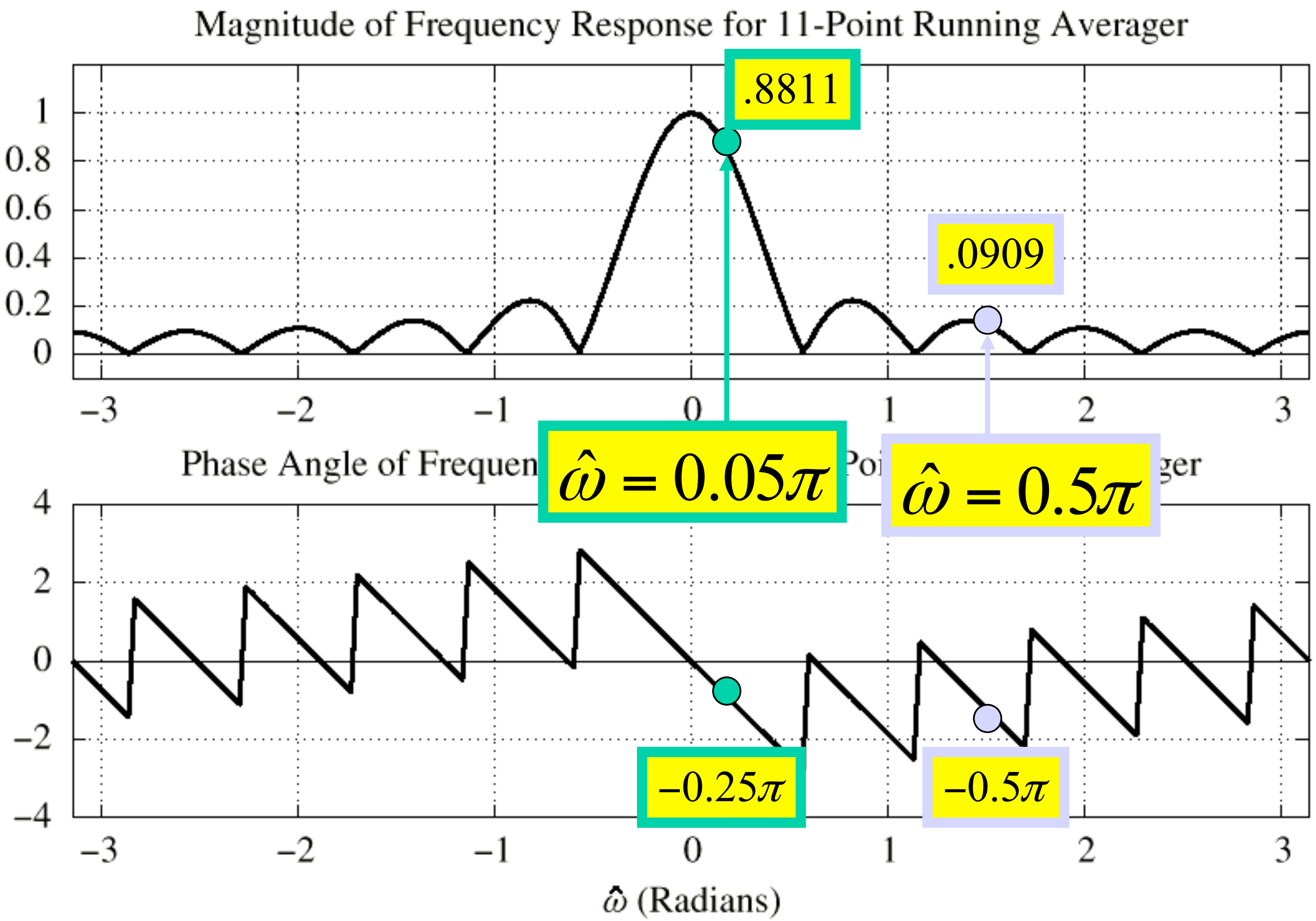
$$x_1(t) = \cos(2\pi(25)t) \quad , \quad T_s = \frac{1}{1000}$$

$$x_1(nT_s) = \cos\left(\frac{2\pi(25)n}{1000}\right) = \cos\left(\frac{\pi}{20}n\right)$$

**NO new freqs**

**WARNING**:
When there is aliasing,
y(t) will have different
frequency components
than x(t)

# FREQUENCY RESPONSE OF 11-pt AVERAGER



# EVALUATE OUTPUT

Input: $x(t) = \cos(2\pi(25)t) + \cos(2\pi(250)t - \frac{1}{2}\pi)$

$H(e^{j0.05\pi}) = 0.8811 e^{-j0.25\pi}$

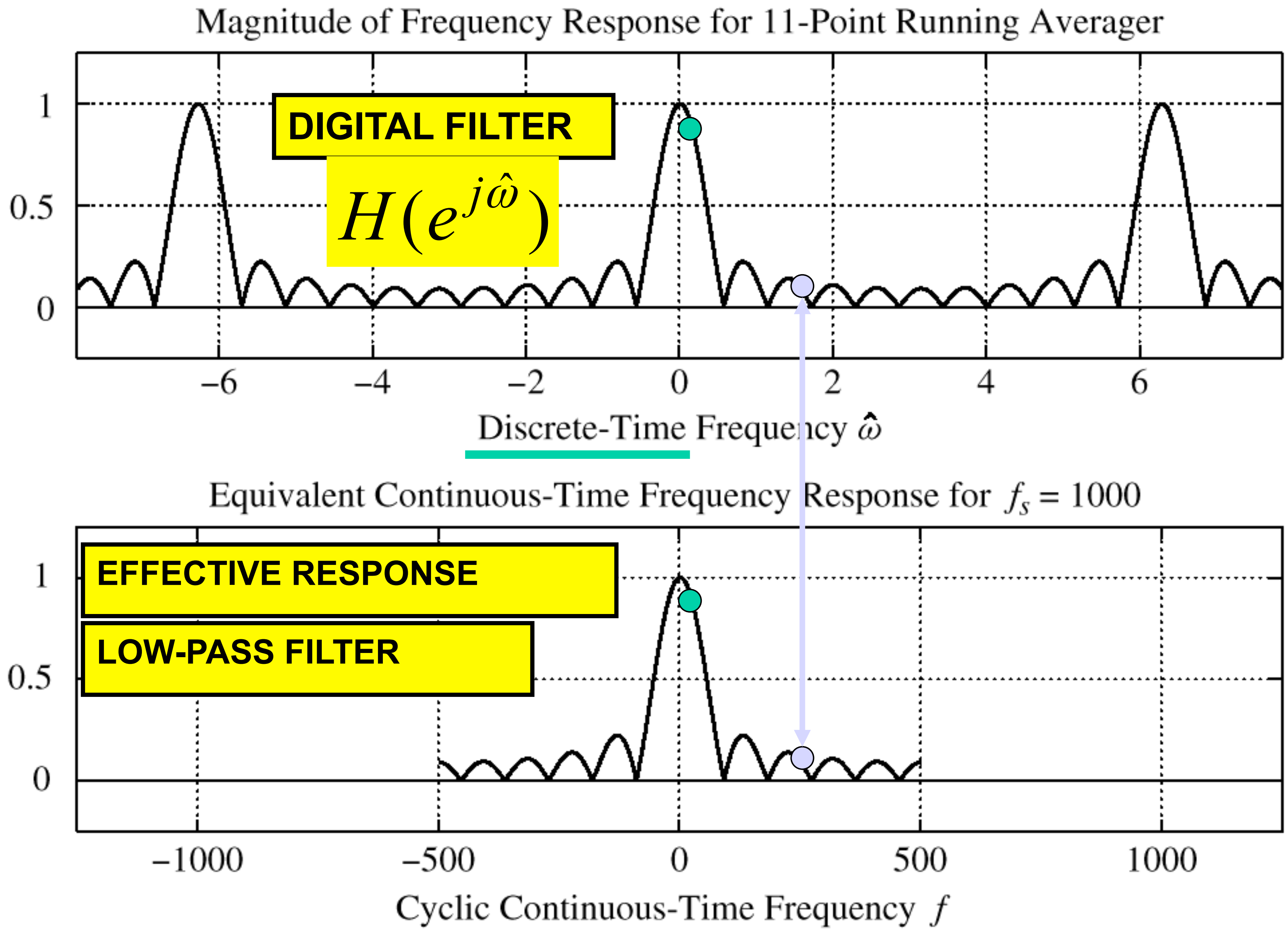
$H(e^{j0.5\pi}) = 0.0909 e^{-j0.5\pi}$

**MAG. SCALE**

**PHASE CHANGE**

Output:

$$y(t) = .8811 \cos\left(2\pi(25)t - \frac{\pi}{4}\right) + .0909 \cos(2\pi(250)t - \pi)$$



Magnitude of Frequency Response for 11-Point Running Averager
DIGITAL FILTER
$H(e^{j\hat{\omega}})$
1
0.5
0
−6
−4
−2
0
2
4
6
Discrete-Time Frequency $\hat{\omega}$
Equivalent Continuous-Time Frequency Response for $f_s = 1000$
EFFECTIVE RESPONSE
LOW-PASS FILTER
1
0.5
0
−1000
−500
0
500
1000
Cyclic Continuous-Time Frequency $f$


- Next week <> Final Exam Review